CHAPTER 5

EXTREME VALUE ANALYSIS

5.1 INTRODUCTION

The purpose of frequency analysis is to analyse past records of hydrologic variables so as to estimate future occurrence probabilities. The data used in the analysis must be evaluated in terms of the objectives, length of records available and completeness of records. It must also satisfy certain statistical criteria such as randomness, independence, homogeneity and stationarity. A frequency analysis can be performed using single-site data, regional data or both. It can also include historical information and reflect physical constraints.

Because hydrological phenomena are characterized by great variability, randomness and uncertainty, it should, therefore be recognized that statistical analysis of hydrological data will not always yield a true answer. The sources of uncertainty in frequency analysis include representativness of the analytical approach, selection of the probability distribution and estimation of parameters.

Hydrological analysis is generally based on well-established principles of hydrodynamics, thermodynamics and statistics. However, the central problem in hydrological analysis is the application of these principles in a natural environment that is non-homogeneous, sparsely sampled and only partially understood. The events sampled are usually unplanned and uncontrolled. Analyses are performed to obtain spatial and temporal information about hydrological variables, regional generalizations and relationships among the variables. Analyses can be performed using deterministic, parametric, probabilistic and stochastic methods. An analysis based on the deterministic approach follows the laws that describe physical and chemical processes. In the parametric approach, an analysis is performed by intercomparison of hydrological data recorded at different locations and times. In the probabilistic approach, the frequency of occurrence of different magnitudes of hydrological variables is analysed. In the stochastic approach, both the sequential order and the frequency of occurrence of different magnitudes are analysed often using time-series methods. Evidence continues to accumulate documenting the dynamic and nonlinear character of the hydrological cycle. In the case of extreme events, our major interest is not in what has occurred, but the likelihood that further extreme and damaging events will occur at some point in the future.

The occurrence of many extreme events in hydrology cannot be forecasted on the basis of deterministic information with sufficient skill and lead time. In such cases, a probabilistic approach is required to incorporate the effects of such phenomena into decisions. If the occurrences can be assumed to be independent in time, in that the timing and magnitude of an event bears no relation to preceding events, then frequency analysis can be used to describe the likelihood of any one or a combination of events over the time horizon of a decision. Hydrological phenomena commonly described by frequency analysis include storm precipitation (5.7), low flows (5.8) and annual flood maxima (5.9).

Both the detail and precision of the analysis should be consistent with the quality and sampling adequacy of the available data and with the accuracy required by the application of the analysis. Consideration should be given to the relationship between the cost and time devoted to an analysis and to the benefits expected. Traditionally, graphical and very simple computational methods have proven more cost effective than more sophisticated methods, and they may be sufficiently accurate for the data and purposes involved. However, the widespread availability of personal computing equipment, with general-purpose statistical software and computation environments such as spreadsheets, has largely replaced hand computational procedures. A major advantage of the modern computational environment is that it should improve an agency's ability to store, retrieve and analyse data. Further, the graphical capabilities of personal computers should greatly enhance the ability of hydrologists to review and understand their data, as well as the results and the computations that they perform.

5.2 STATISTICAL SERIES AND RETURN PERIODS [HOMS H83]

In frequency analysis, a series is a convenient sequence of data, such as hourly, daily, seasonal or annual observations of a hydrological variable. If the record of these observations contains all the events that occurred within a given period, the series is called a complete duration series. For convenience, the record often contains only events of magnitude above a pre-selected base or threshold level; this is called a partial duration series or peaks-over-threshold series. A series that contains only the event with the largest magnitude that occurred in each year is called an annual maximum series.

The use of the annual maximum series is very common in frequency analyses for two reasons. The first is for convenience, as most data are processed in such a way that the annual series is readily available. The second is that there is a simple theoretical basis for extrapolating the frequency of annual series data beyond the range of observation. With partial series data, such theory is not as simple because one must consider the arrival process of floods within a year and the distribution of the magnitude of floods when they do occur. Another problem with partial duration series is the lack of independence of events that might follow one another in close sequence, as well as seasonal effects. However, if the arrival rate for peaks over the threshold is large enough and can be modelled by simple two-parameter distributions, for example 1.65 for the Poisson arrival with exponential exceedances model, it should yield more accurate estimates of flood quantiles than the corresponding annual flood frequency analyses. However, when fitting a three-parameter distribution, such as the generalized Pareto distribution for exceedances with Poisson arrivals, there appears to be no advantage in using a partial duration series no matter how many floods are recorded on average each year (Martins and Stedinger, 2000). It should not be a surprise that recording the value of a great many small events does not tell us much about the risk of very large events occurring unless the structure of the model is fairly rigid.

A limitation of annual series data is that each year is represented by only one event. The second highest event in a particular year may be higher than the highest in some other years, yet it would not be contained in the series. The use of partial duration series can address this issue because all peaks above the specified threshold are considered.

The complete duration series may be required for the stochastic approach in which independence is not required. It may also serve for a probabilistic analysis of data from arid regions where the events are rare and almost independent.

The return period T of a given level is the average number of years within which the event is expected to be equalled or exceeded only once. The return period is equal to the reciprocal of the probability of exceedance in a single year. If the annual exceedance probability is denoted $1/T_a$, where T_a is the annual return period, the relationship between the annual return period and the return period in the partial duration series can be expressed as follows:

$$1/T_a = 1 - exp\{- \lambda\, q_e\} = 1 - exp\{- l/T_p\} \quad (5.1)$$

where $T_p = 1/(\lambda\ qe)$ is the average return period in the partial duration series with λ being the arrival rate for peaks over the threshold and q_e is the probability that when such a flood occurs, it exceeds the level of concern. This equation can be solved for T_p to obtain:

$$T_p = 1 / ln\ [1 - 1/T_a] \quad (5.2)$$

T_p is less than T_a because more than one event can occur per year in a partial duration series. For return periods exceeding ten years, the differences in return periods obtained with the annual and partial series is inconsequential. Table II.5.1 compares the return periods for an annual maximum series and a partial duration series. This formula is based on the assumption that floods in the partial duration series occur independently in time and at a constant rate; relaxation of that assumption yields different relationships (Robson and Reed, 1999). NERC (1975) observes that the actual probabilistic model for arrivals with regard to large return period events is not particularly important, provided that different models yield the same average number of arrivals per year (see also Cunnane, 1989).

Table II.5.1. Corresponding return periods for annual and partial series

Partial series	*Annual series*
0.50	1.16
1.00	1.58
1.45	2.00
2.00	2.54
5.00	5.52
10.00	10.50

5.3 PROBABILITY DISTRIBUTIONS USED IN HYDROLOGY [HOMS H83, X00]

Probability distributions are used in a wide variety of hydrological studies, including studies of extreme high and low flows, droughts, reservoir volumes, rainfall quantities and in time-series models. Table II.5.2 lists the most commonly used distributions in hydrology. Their mathematical definitions are given in a number of references (Kite, 1988; Cunnane, 1989; Bobee and Ashkar, 1991; Stedinger and others, 1993; Clark, 1994; Kottegoda and Rosso, 1997 and Hosking and Wallis, 1997).

Numerous probability distributions have been introduced in the literature to model hydrological phenomena such as extreme events. Despite intensive research and study, no particular model is considered superior for all practical applications. The user should, therefore, screen available models in the light of the problem to be solved and the nature of the available data. Consequently, only some distributions that are in common use are reviewed in this chapter. The contending distributions that fit the observed data satisfactorily usually differ significantly in the tail of the distribution, especially when extrapolation is involved. No general guidance is available for extrapolating distributions, particularly beyond twice the available record length. The decision regarding which distribution to use should be based on the comparison of the suitability of several candidate distributions. The advantages and disadvantages of the various methods that can be used for this objective are discussed in 5.6.

Annual totals, such as flow volumes or rainfall depths, tend to be normally distributed or almost so because of the forces described by the central limit theorem of statistics. Monthly and weekly totals are less symmetric, displaying a definite skewness that is mostly positive and cannot usually be modelled by the normal distribution. Annual extremes – high or low – and peaks over a threshold tend to have skewed distributions. The part of a sample that lies near the mean of the distribution can often be described well by a variety of distributions. However, the individual distributions can differ significantly and very noticeably from one another in the values estimated for large return periods, as well as very small cumulative probabilities. As hydraulic design is often based on estimates of large recurrence-interval events, it is important to be able to determine them as accurately as possible. Hence, the selection of the distribution is very important for such cases. The choice of distributions is discussed in the references cited above, which include discussions on the methods available for choosing between distributions. This is also discussed in 5.6.

Generally, mathematical distributions having three parameters, such as those appearing in Table II.5.2, are selected so as to make the distribution matches the available data more consistently. In some cases an empirical distribution can be used to describe the data, thereby avoiding the use of mathematical parametric distributions.

Use of a mathematical distribution has several advantages:

(a) It presents a smooth and consistent interpretation of the empirical distribution. As a result, quantiles and other statistics computed using the fitted distribution should be more accurate than those computed with the empirical distribution;
(b) It provides a compact and easy-to-use representation of the data;
(c) It is likely to provide a more realistic description of the range and likelihood of values that the random variable may assume. For example, by using the empirical distribution, it is implicitly assumed that no values larger or smaller than the sample maximum or minimum can occur. For most situations this is entirely unreasonable.

There are several fundamental issues that arise in selecting a distribution for frequency analysis (Stedinger and others, 1993):

(a) What is the true distribution from which the observations are drawn?
(b) Is a proposed flood distribution consistent with available data for a particular site?
(c) What distribution should be used to obtain reasonably accurate and robust estimates of flood quantiles and flood risk for hydrological design purposes?

Unfortunately, the answer to the first question will never be known, and it might not be much help if it were. The true distribution of the data could be incredibly complex with more parameters than a hydrologist could ever hope to estimate. Thus, the aim is to establish a good, but simple approximation of the true distribution of the events. Standard goodness-of-fit statistics and probability plots can, at least in part, address the second question, as they will sometimes show that particular distributions are not consistent with the available data. There may be pragmatic considerations to prevent the use of a distribution for a particular sample. For example, the distribution may be upper bounded at what is considered to be an unreliable low value, thereby not providing an acceptable estimate of extreme conditions. As a practical matter, many national agencies look at the problem from the point of view of the third question: What distribution coupled with a reasonable fitting procedure will yield good estimates of risk in their region of the world? Thus, the aim is not to seek absolute truths. Instead, the goal is to develop practical procedures which, with the data in hand or data that can be collected, will provide a good approximation of the frequency relationships of interest. Over the past four decades, various distributions have been introduced for use in hydrological frequency analysis. The following section provides an overview of some of these distributions.

5.3.1 Normal family: N, LN and LN3

5.3.1.1 Normal distribution

The normal distribution (N) is useful in hydrology for describing well-behaved phenomena, such as the total annual flow. The probability density function for a normal random variable X is given in Table II.5.2, and it is unbounded both above and below, with mean μ_x and variance σ^2_x. The normal distribution's skewness coefficient is zero, because the distribution is symmetric. The cumulative distribution function (CDF) of the normal distribution is not available in closed form, but books on statistics include tables of the standardized normal variate z_p. The quantity z_p is an example of a frequency factor because the p^{th} quantile x_p of any normal distribution with mean μ and variance σ^2 can be written as follows:

$$x_p = \mu + \sigma z_p \tag{5.3}$$

5.3.1.2 Log-normal distribution

In general, flood distributions are positively skewed and not properly described by a normal distribution. In many cases the random variable corresponding to the logarithm of the flood flows will be adequately described by a normal distribution. The resulting two-parameter log-normal (LN) distribution has the probability-density function given in Table II.5.2. Often, the logarithms of a random variable X are not distributed normally. In such cases, introducing a boundary parameter ζ before taking logarithms can solve this problem, yielding a three-parameter log-normal distribution (LN3) (Stedinger and others, 1993) so that:

$$Y = ln\,[X - \zeta] \tag{5.4}$$

would have a normal distribution. Thus:

$$X = \zeta + \exp\,(Y) \tag{5.5}$$

has a LN3 distribution. In terms of the frequency factors of the standard normal distribution z_p, the quantiles of a log-normal distribution are as follows:

$$x_p = \zeta + \exp\,(\mu_Y + \sigma_Y z_p) \tag{5.6}$$

where μ_Y and σ_Y are the mean and standard deviation of Y. Parameter estimation procedures are compared by Stedinger (1980).

5.3.2 Extreme value distributions: Gumbel, generalized extreme value and Weibull

Gumbel (1958) defined three types of extreme value distributions which should describe the distribution of the largest or smallest value in a large sample. They have been widely used in hydrology to describe the largest flood or the lowest flow.

5.3.2.1 Gumbel distribution

Annual floods correspond to the maximum of all of the flood flows that occur within a year. This suggests their distribution is likely to be a member of a general class of extreme value (EV) distributions developed in Gumbel (1958). Let $X_1,\ldots,X_n$ be a set of annual maximum discharges and let $X = max\{X_i\}$. If the X_i are independent and identically distributed random variables unbounded above, with an exponential-like upper tail, then for large n the variate X has an extreme value (EV) type I distribution or Gumbel distribution with cumulative distribution function given in Table II.5.2.

Landwehr and others (1979) and Clarke (1994) discuss estimation procedures and Hosking (1990) has shown that L–moments provide accurate quantile estimates for the small sample sizes typically available in hydrology.

5.3.2.2 Generalized extreme value distribution

The generalized extreme value distribution spans the three types of extreme value distributions for maxima. The Gumbel and generalized extreme value distribution distributions are widely used for flood frequency analyses around the world (Cunnane, 1989). Table II.5.2 provides the cumulative distribution function of the generalized extreme value distribution.

Table II.5.2. Commonly used frequency distributions (after Stedinger and others, 1993)

Distribution	*Probability density function and/or cumulative distribution function*	*Range*	*Moments*
Normal	$f_X(x) = \frac{1}{\sqrt{2\pi\sigma_X^2}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu_X}{\sigma_X}\right)^2\right]$	$-\infty < x < \infty$	μ_X and σ^2_X, $\gamma_X = 0$
Log-normal[a]	$f_X(x) = \frac{1}{x\sqrt{2\pi\sigma_Y^2}} \exp\left[-\frac{1}{2}\left(\frac{\ln(x)-\mu_Y}{\sigma_Y}\right)^2\right]$	$0 < x$	$\mu_X = \exp[\mu_Y + \sigma^2_Y/2]$ $\sigma^2_X = \mu^2_X \{\exp[\sigma^2_Y] - 1\}$ $\gamma_X = 3CV_X + CV^3_X$
Pearson type III	$f_X(x) = \lvert\beta\rvert\, [\beta(x-\xi)]^{\alpha-1} \exp[-\beta(x-\xi)]/\Gamma(\alpha)$ (for $0 < \beta$ and $\xi = 0$: $\gamma_X = 2\,(CV_X)$)	$0 < \alpha$ for $0 < \beta : \xi < x$ for $\beta < 0$: $x < \xi$	$\mu_X = \xi + \alpha/\beta$; $\sigma^2_X = \alpha/\beta^2$ and $\gamma_X = 2/\sqrt{\alpha}$ and $\gamma_X = -2/\sqrt{\alpha}$
Log-Pearson type III	$f_X(x) = \lvert\beta\rvert\, \{\beta[\ln(x)-\xi]\}^{\alpha-1} \exp\{-\beta[\ln(x)-\xi]\}/x\Gamma(\alpha)$ for $\beta < 0$, $0 < x < \exp(\xi)$; for $0 < \beta$, $\exp(\xi) < x < \infty$	See Stedinger and others (1993).	
Exponential	$f_X(x) = \beta \exp\{-\beta(x-\xi)\}$ $F_X(x) = 1 - \exp\{-\beta(x-\xi)\}$	$\xi < x$ for $0 < \beta$	$\mu_X = \xi + 1/\beta$; $\sigma^2_X = 1/\beta^2$ $\gamma_X = 2$
Gumbel	$f_X(x) = (1/\alpha) \exp\{-(x-\xi)/\alpha - \exp[-(x-\xi)/\alpha]\}$ $F_X(x) = \exp\{-\exp[-(x-\xi)/\alpha]\}$	$-\infty < x < \infty$	$\mu_X = \xi + 0.5772\,\alpha$ $\sigma 2^2_X = \pi^2\alpha^2/6 = 1.645\alpha^2$; $\gamma_X = 1.1396$
Generalized extreme value	$F_X(x) = \exp\{-[1 - \kappa(x-\xi)/\alpha]^{1/\kappa}\}$ when $0 < \kappa$, $x < (\xi+\alpha/\kappa)$; $\kappa < 0$, $(\xi+\alpha/\kappa) < x$	(σ_X^2 exists for $-0.5 < \kappa$)	$\mu_X = \xi + (\alpha/\kappa)\,[1 - \Gamma(1+\kappa)]$ $\sigma^2_X = (\alpha/\kappa)^2 \{\Gamma(1+2\kappa) - [\Gamma(1+\kappa)]^2\}$
Weibull	$f_X(x) = (k/\alpha)\,(x/\alpha)^{k-1} \exp[-(x/\alpha)^k]$ $F_X(x) = 1 - \exp[-(x/\alpha)^k]$	$0 < x$; $0 < k, \alpha$	$\mu_X = \alpha\,\Gamma(1 + 1/k)]$ $\sigma^2_X = \alpha^2 \{\Gamma(1 + 2/k) - [\Gamma(1 + 1/k)]^2\}$
Generalized logistic	$y = [1 - \kappa(x-\xi)/\alpha]^{1/\kappa}$ for $\kappa \neq 0$ $f_X(x) = (1/\alpha)\,[y^{(1-\kappa)}/(1+y)]^2$ $F_X(x) = 1/[1 + y]$	$y = \exp[-(x-\xi)/\alpha]$ for $\kappa = 0$ for $\kappa < 0$, $\xi + \alpha/\kappa \le x < \infty$ for $0 < \kappa$, $-\infty < x \le \xi + \alpha/\kappa$	$\mu_X = \xi + \alpha/[1/\kappa - \pi/\sin(\kappa\pi)]$ See Ahmad and others (1998) for σ^2_X.
Generalized Pareto	$f_X(x) = (1/\alpha)\,[1 - \kappa(x-\xi)/\alpha]^{1/\kappa-1}$ $F_X(x) = 1 - [1 - \kappa(x-\xi)/\alpha]^{1/\kappa}$	for $\kappa < 0$, $\xi \le x < \infty$ for $0 < \kappa$, $\xi \le x \le \xi + \alpha/\kappa$ (γ_X exists for $\kappa > -0.33$)	$\mu_X = \xi + \alpha/(1+\kappa)$ $\sigma^2_X = \alpha^2/[(1+\kappa)^2(1+2\kappa)]$ $\gamma_X = 2(1-\kappa)(1+2\kappa)^{1/2}/(1+3\kappa)$
Halphen			
Type A	$f_X(x) = \frac{1}{2m^\nu K_\nu(2\alpha)}\, x^{\nu-1} \exp\left[-\alpha\left(\frac{x}{m} + \frac{m}{x}\right)\right]$	for $x > 0$; $m > 0$; $\alpha > 0$; $-\infty < \alpha < \infty$[b]	
Type B	$f_X(x) = \frac{2}{m^{2\nu} ef_\nu(\alpha)}\, x^{2\nu-1} \exp\left[-\left(\frac{x}{m}\right)^2 + \alpha\left(\frac{x}{m}\right)\right]$	for $x > 0$; $m > 0$; $\nu > 0$; $-\infty < \alpha < \infty$[c]	See Marlat (1956).
Type B[–1]	$f_X(x) = \frac{2m^{2\nu}}{ef_\nu(\alpha)}\, x^{-2\nu-1} \exp\left[-\left(\frac{m}{x}\right)^2 + \alpha\left(\frac{m}{x}\right)\right]$	for $x > 0$; $m > 0$; $\nu > 0$; $-\infty < \alpha < \infty$[c]	

[a] Here $Y = \ln(X)$. A three-parameter log-normal distribution with $Y = \ln(X - \xi)$ is also commonly used.
[b] K_ν = modified Bessel function, second kind.
[c] $ef_\nu(\alpha)$ = exponential factorial function.

The Gumbel distribution is a special case of generalized extreme value distribution corresponding to $\kappa = 0$. Here, x is a location parameter, α is a scale parameter, and κ is the important shape parameter. For $\kappa > 0$ the distribution has a finite upper bound at $\xi + \alpha/\kappa$; for $\kappa < 0$, the distribution has a thicker right-hand tail and is unbounded above.

Hosking and others (1985) describe the L–moment procedure that is effective with this distribution. L–moments have been the basis of many regional and index-flood procedures that make use of the generalized extreme value distribution (Hosking and Wallis, 1997). More recently, Martins and Stedinger (2000) present generalized maximum likelihood estimators for the generalized extreme value distribution that are more accurate than L–moment estimators over the range of hydrological interest.

5.3.2.3 Two-parameter Weibull distribution

If W_i are the minimum streamflow in different days of the year, then the annual minimum is the smallest of the W_i, each of which is bounded below by zero. In this case the random variable $X = \min \{W_i\}$ may be described well by the extreme value type III distribution for minima, or the Weibull distribution (see Figure II.5.1 and Table II.5.2). For $k < 1$, the Weibull probability density goes to infinity as x approaches zero, and decays slowly for large values of x. For $k = 1$, the Weibull distribution reduces to the exponential distribution corresponding to $\gamma = 2$. For $k > 1$, the Weibull density function is like a density function of Pearson type III distribution for small values of x and $\alpha_{P3} = k$, but decays to zero faster for large values of x. Parameter estimation methods are discussed in Kite (1988).

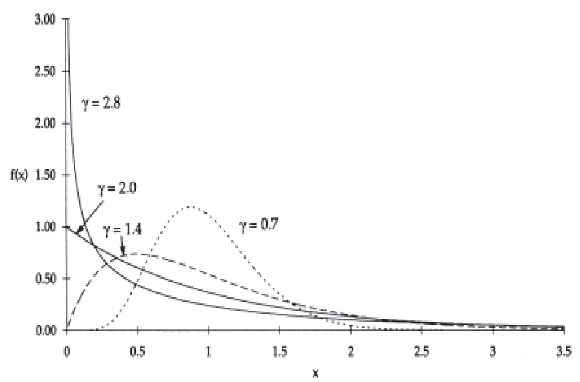


Figure II.5.1. The probability density function for the Pearson type III distribution with lower bound $\zeta = 0$, mean $\mu = 1$ and coefficients of skewness $\gamma = 0.7, 1.4, 2.0$ and 2.8 (corresponding to a gamma distribution and shape parameters $\alpha = 8, 2, 1$ and 0.5, respectively)

5.3.3 Pearson type III family

The Pearson type III (P3) distributions are commonly used to fit a sample of extreme hydrological data. A theoretical description of this distribution can be found in Bobée and Ashkar (1991) and a summary in Maidment's *Handbook of Hydrology*, Chapter 18 (Stedinger and others, 1993). The notations of that publication are used in the following. The probability density function of the P3 distribution, given in Table II.5.2, is defined by three parameters: ζ (location), β (scale) and α (shape). The method of moments considering mean, variance and coefficient of skewness is used by the Interagency Advisory Committee on Water Data (1982) to fit the P3 distribution to data. Caution should be exercised in using moments, as they may yield an upper bound which might be smaller than an observed flood. The method of maximum likelihood can also be used (Pilon and Harvey, 1992). This distribution can be used for both positively and negatively skewed samples.

The log-Pearson type III distribution (LP3) describes a variable x whose logarithm $y = \log x$ is P3 distributed. This distribution was recommended for the description of floods in the United States of America by the United States Water Resources Council, initially in 1966 and then again by the Interagency Advisory Committee on Water Data in 1982. It was also adopted for use in Canada as one of several other methods (Pilon and Harvey, 1992).

5.3.4 Halphen family: types A, B and B^{-1}

This family of distributions was specifically designed to model floods and more generally, extremes. The probability density function of these distributions (Perreault and others, 1999*a*) are given in Table II.5.2. Perreault and others (1999*b*) presented procedures for estimating parameters, quantiles and confidence intervals for the Halphen distributions. The Gamma and inverse Gamma (x is the inverse Gamma distributed if $y = 1/x$ follows Gamma distributions) are limiting cases of the Halphen distributions.

Although the probability density function of the Halphen distributions are mathematically more complicated than the three-parameter distributions currently used in hydrometeorology, that should not be a serious obstacle for their use in practice, since the Halphen distributions can be applied with the aid of user-friendly software such as HYFRAN (www.ete.inrs.ca/activites/groupes/chaire_hydrol/hyfran.html).

5.3.5 Generalized logistic distribution

The generalized logistic distribution was introduced to the mainstream of the hydrological literature by Hosking and Wallis (1997) and was proposed as the distribution for flood frequency analysis in the United Kingdom (Robson and Reed, 1999). The parameterization is similar to the generalized extreme value distribution, and both have Pareto-like tails for large values of x. The cumulative distribution function of the generalized logistic distribution is given in Table II.5.2, as is the range of the variable. Hosking and Wallis (1997) and Robson and Reed (1999) document how the three parameters of this distribution can be obtained from L–moment estimators.

5.3.6 Generalized Pareto distribution

The generalized Pareto distribution has a very simple mathematical form (see Table II.5.2) and is useful for modelling events that exceed a specified lower bound at which the density function has a maximum ($\kappa < 1$). Examples include daily rainfall depths and all floods above a modest threshold. Hosking and Wallis (1987) discuss alternative estimation procedures. Often the value of the lower bound can be determined by the physical constraints of the situation, so that only two parameters need be estimated. If the physical situation does not dictate the value of the lower bound, then the smallest observation may suffice as an estimator of the lower bound for x.

A very interesting relationship exists between the generalized Pareto distribution and the generalized extreme value distribution. If peaks in a partial duration series arrive as in a Poisson process and have magnitudes described by a generalized Pareto distribution, then the annual maxima greater than the partial duration series threshold have a generalized extreme value distribution with the same value of κ (Stedinger and others, 1993). Wang (1991) and Martins and Stedinger (2001) explore the relative efficiency of the two modelling frameworks.

5.3.7 Non-parametric density estimation method

The non-parametric method does not require either the assumption of the functional form of the overall density function, or the estimation of parameters based on the mean, variance and skew. The non-parametric kernel density estimation requires the selection of a kernel function K, which is a probability density function, and the calculation of a smoothing factor H. Then, using a sample of N observations of the variable x, an approximation of the probability density function for the variable x is obtained by assigning each x_j a probability of $1/N$ and then using the kernel function to spread out that probability around the value of each x_j to obtain the following equation:

$$f(x) = \frac{1}{NH} \sum_{i=1}^{N} K\left(\frac{x - x_i}{H}\right) \tag{5.7}$$

The principle of a kernel estimator as expressed by the above equation is that a kernel of prescribed form, triangular, normal, or Gumbel distribution function is associated with each observation over a specified scale, expressed by H. The weighted sum of these functions constitutes the non-parametric estimate of the density function. The optimal value of H can be determined based on a cross-validation procedure (Adamowski, 1985) and is available in a computer software package (Pilon and others, 1992).

5.4 HYPOTHESIS TESTING

The data series must meet certain statistical criteria such as randomness, independence, homogeneity and stationarity in order for the results of a frequency analysis to be theoretically valid. These statistical criteria are explained in Table II.5.3, where appropriate statistical tests are indicated. A more detailed description of many of these tests can be found in Helsel and Hirsch (1992). Well-known statistical parametric tests such as the t-test and the F-test are not included in the table because hydrological data series often do not satisfy some conditions for strict applicability of these tests, particularly the assumption of normality, which can adversely impact upon the power of parametric tests (Yue and Pilon, 2004). The tests indicated in the table are of a non-parametric type, which avoids assumptions regarding the underlying parametric distribution of the data. Care should be taken to verify the assumptions underlying the tests, as violation may lead to unreliable results (Yue and others, 2002*a*).

Statistical tests can only indicate the significance of the observed test statistics and do not provide unequivocal findings. It is therefore important to clearly understand the interpretation of the results and to corroborate findings with physical evidence of the causes, such as land use changes. When data do not satisfy the assumptions, then a transformation can often be employed so that the transformed

Table II.5.3. Statistical tests and statistical criteria (after Watt, 1989)

Criterion	*Explanation*	*Applicable statistical tests*
Randomness	In a hydrologic context, randomness means essentially that the fluctuations of the variable arise from natural causes. For instance, flood flows appreciably altered by reservoir operation are unnatural and therefore cannot be considered as random, unless the effect of the regulation is removed first.	No suitable tests for hydrological series are available.
Independence	Independence implies that no observation in the data series has any influence on any following observations. Even if events in a series are random, they may not be independent. Large natural storages, in a river basin, for example, may cause high flows to follow high flows and low flows to follow low flows. The dependence varies with the interval between successive elements of the series: dependence among successive daily flow values tends to be strong, while dependence between annual maximum values is generally weak. Likewise, the elements of annual series of short-duration rainfall may, in practice, be assumed to be independent. In some cases, however, there may be significant dependence even between annual maximum values, for example in the case of rivers flowing through very large storages such as the Great Lakes of North America.	– Anderson as described in Chow (1964). – Spearman rank order serial correlation coefficient as described in NERC (1975).
Homogeneity	Homogeneity means that all the elements of the data series originate from a single population. Elderton (1953) indicated that statistics are seldom obtained from strictly homogeneous material. For instance, a flood series that contains both snowmelt and rainfall floods may not be homogeneous; however, depending on the results of a test, it may be acceptable to treat it as such. When the variability of the hydrological phenomenon is too high, as in the case of extreme precipitation, non-homogeneity tends to be difficult to decipher (Miller, 1972), but non-homogeneity in yearly precipitation sums is easier to detect.	Terry (1952).
Stationarity	Stationarity means that, excluding random fluctuations, the data series is invariant with respect to time. Types of non-stationarity include trends, jumps and cycles. In flood analysis, jumps are generally due to an abrupt change in a basin or river system, such as the construction of a dam. Trends may be caused by gradual changes in climatic conditions or in land use , such as urbanization. Cycles may be associated with long-term climatic oscillations.	– Spearman rank correlation coefficient test for trend (NERC, 1975) – Wald–Wolfowitz (1943) test for trend. No satisfactory method of testing is available for long-period cycles. – Mann–Kendall test for trend (Yue and others, 2002*b*)

observations would meet the criteria required for analysis. Caution is advised in interpolation and extrapolation when data do not meet the assumptions.

5.4.1 Wald–Wolfowitz test for independence and stationarity

Given the data sample of size N $(x_1,\ldots,x_N)$, the Wald–Wolfowitz test considers the statistic R so that:

$$R = \sum_{i=1}^{N-1} x_i x_{i+1} + x_1 x_N \qquad (5.8)$$

When the elements of the sample are independent, R asymptotically follows asymptotically normal distribution with mean and variance given by the following equations:

$$\bar{R} = (s^2_1 - s_2) / (N - 1) \qquad (5.9)$$

$$Var(R) = (s^2_2 - s_4) / (N - 1) - \bar{R}^2 \qquad (5.10)$$
$$+ (s_1^4 - 4s_1^2 s_2 + 4s_1 s_3 + s_2^2 - 2s_4) / (N - 1)(N - 2)$$

with $s_r = Nm'_r$ and m'_r is the r^th^ moment of the sample about the origin.

The quantity $(R - \bar{R}) / (Var\,(R))^{1/2}$ follows a standardized normal distribution (mean 0 and variance 1) and can be used to test at level α the hypothesis of independence by comparing $|n|$ with the standard normal variate $u_{\alpha/2}$ corresponding to a probability of exceedance $\alpha/2$.

5.4.2 Mann–Kendall test for trend detection

The Mann–Kendall test is a rank-based non-parametric test for assessing the significance of a trend. The null hypothesis H_0 is that a sample of data ordered chronologically is independent and identically distributed. The statistic S is defined as follows (Yue and others, 2002*b*):

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \mathrm{sgn}(x_j - x_i) \tag{5.11}$$

where

$$\mathrm{sgn}\,(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases} \tag{5.12}$$

When $n \geq 40$, the statistic S is asymptotically normally distributed with mean 0 and variance given by the following equation:

$$Var\left\{S\right\} = \frac{1}{18}\left[n(n-1)(2n+5) - \sum_t t(t-1)(2t+5)\right] \tag{5.13}$$

where t is the size of a given tied group and $\sum_t$ is the summation over all tied groups in the data sample. The standardized test statistic K is computed by using the following equation:

$$K = \begin{cases} \dfrac{S-1}{\sqrt{Var\,(S)}} & \text{If S} > 0 \\ 0 & \text{If S} = 0 \\ \dfrac{S+1}{\sqrt{Var\,(s)}} & \text{If S} < 0 \end{cases} \tag{5.14}$$

The standardized statistic K follows the standard normal distribution with mean zero and variance of one. The probability value P of the statistic K of sample data can be estimated using the normal cumulative distribution function as:

$$P = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2} dt \tag{5.15}$$

For independent sample data without trend, the P value should be equal to 0.5. For sample data with large positive trend, the P value should be close to 1.0, whereas a large negative trend should yield a P value close to 0.0. If the sample data are serially correlated, then the data should be pre-whitened and a correction applied to calculate the variance (Yue and others, 2002*b*).

The slope of a trend is estimated as follows:

$$\beta = median\left(\frac{x_i - x_j}{i - j}\right), \forall j < i \tag{5.16}$$

where β is the estimate of the slope of the trend and x_j is the j^{th} observation. An upward trend is represented by a positive value of β and a downward trend is represented by a negative value of β.

5.4.3 Mann–Whitney test for homogeneity and stationarity (jumps)

We now consider two samples of size p and q (with $p \leq q$) the combined set of size $N = p + q$ is ranked in increasing order. The Mann–Whitney test considers the following quantities:

$$V = R - p(p+1) / 2 \tag{5.17}$$

$$W = pq - V \tag{5.18}$$

where R is the sum of the ranks of the elements of the first sample of size p in the combined series and V and W are calculated from R, p and q. V represents the number of times that an item in sample 1 follows in the ranking an item in sample 2; W can also be computed in a similar way for sample 2 following sample 1.

The test statistic, U, is defined by the smaller of V and W. When $N > 20$, and p, $q > 3$, and under the null hypothesis that the two samples come from the same population, U is approximately normally distributed with mean:

$$\bar{U} = pq/2 \tag{5.19}$$

and variance:

$$Var(U) = \left[\frac{pq}{N(N-1)}\right]\left[\frac{N^3 - N}{12} - \sum T\right] \tag{5.20}$$

with $T = (J^3 - J) / 12$, where J is the number of observations tied at a given rank. The summation ΣT is over all groups of tied observations in both samples of size p and q. For a test at a level of significance, the quantity $|u| = |(U - \bar{U})/Var(U)^{1/2}|$ is compared with the standardized normal quantile $u_{\alpha/2}$ corresponding to a probability of exceedance $\alpha/2$.

5.4.4 Sample size and length of record

The definition of a stable distribution for estimating future probabilities of occurrence of a hydrological phenomenon requires that the length of record or sample size must be sufficiently long. In estimating daily extreme precipitation, Sevruk and Geiger (1981) report that the length of record needed to obtain a stable distribution is related to the general humidity of the region and its physiographic conditions that determine the variability of the daily precipitation sum. As indicated in Table II.5.3, when the variability of the hydrological phenomenon is too high, difficulties in testing the homogeneity of the hydrological series can arise. When the coefficient of variation of a sample drawn from a skewed distribution is large (large variability), the standard error of the sample coefficient of skewness which is used to fit the assumed distribution will also be large. Sevruk and Geiger (1981) argue that for extreme precipitation frequency analysis a 25-year period of record may be sufficient in humid regions such as the northern Russian Federation, but even a 50-year period is not adequate in other regions where a distinct periodic fluctuation of precipitation exists. According to these authors, a record of 40 to 50 years is, in general, satisfactory for extreme precipitation frequency analysis. Yue and others (2002*a*) and Yue and Pilon (2004) show, as well, how statistical characteristics of the sample and record length can impact upon the power of common statistical tests.

5.4.5 Grubbs and Beck test for detection of outliers

An outlier is defined as a data point that is far from the bulk of the data. The presence of outliers in a data sample can cause difficulties when attempting to fit a distribution to the sample. There may exist high or low outliers, or both, in a sample, and these can have different impacts on the frequency analysis. Although the problem of treating outliers is still subject to much discussion, certain procedures have been used in hydrology for their identification and treatment, such as those described by the United States Water Resources Council (1981) for flood frequency analysis or by Sevruk and Geiger (1981) for extreme precipitation.

The Grubbs and Beck test for the detection of outliers is the test that is recommended by the United States Water Resources Council (1981). To apply this test, the assumption must be made that the logarithms or some other function of the hydrological series are normally distributed because the test is applicable only to samples from a normal population. It is common to make the simple assumption used by the United States Water Resources Council that the logarithms of the sample values are normally distributed. To apply the Grubbs and Beck test, the following two quantiles are calculated:

$$X_H = exp\ (\bar{x} + K_N s) \tag{5.21}$$

$$X_L = exp\ (\bar{x} + K_N s) \tag{5.22}$$

where $\bar{x}$ and s are the mean and standard deviation of the natural logarithms of the sample, respectively, and K_N is the Grubbs and Beck statistic tabulated for various sample sizes and significance levels. At the 10 per cent significance level, the following polynomial approximation proposed by Pilon and Harvey (1992) can be used for estimating the tabulated values:

$$K(N) = -3.62201 + 6.2844N^{1/4} - 2.49835N^{1/2} + 0.491436N^{3/4} - 0.037911N \tag{5.23}$$

where N is the sample size. In applying the Grubbs and Beck test, any sample values greater than X_H are considered to be high outliers and those less than X_L are considered to be low outliers. For $5 \leq N \leq 150$, $K(N)$ can be computed from the following equation (Stedinger and others, 1993):

$$K(N) = -0.9043 + 3.345\sqrt{log\ (N)} - 0.4046\ log\ (N) \tag{5.24}$$

5.4.6 Bayesian procedures

While the frequency estimation of probability is based on the idea of an experiment that can be repeated several times, the Bayesian approach is based on a personal assessment of probability and provides an opportunity to take into account any information that is available, by means of the prior distribution. Unlike classical models, Bayesian models consider the parameters of the problem as random variables rather than fixed values. For example, in the case of the detection of shifts in the mean of a time series, classical statistical methods assume knowledge of the time of the possible shift. The Bayesian approach, however, does not make any assumptions concerning knowledge of the time of the shift. This allows the approach to make inferences on its characteristics, such as the change point and the amount of shift.

Perreault and others (1999*c*) and other authors have presented Bayesian models for the detection of a single shift in the mean. Perreault and others (2000) presented a method for a change in variability and applied it to hydrological data, while Asselin and others (1999) presented a bivariate Bayesian model for the detection of a systematic change in the mean. A complete description of the Bayesian statistical inference theory is presented in Box and Tiao (1973).

5.5 POPULATION STATISTICS AND PARAMETER ESTIMATION

Assuming that extreme events are described properly by some family of distributions, a hydrologist's task is to estimate the parameters of the distribution so that required quantiles and expectations can be calculated with the fitted model. The statistical and hydrological literature contains many methods and philosophies for estimating the parameters of different distributions: those most commonly employed are outlined below.

5.5.1 Parameter calculation methods

Perhaps the simplest approach is the method of moments, which computes estimates of the parameters so that the theoretical moments of a distribution match the computed sample moments. The recommended procedure for federal agencies in the United States (Thomas, 1985; Interagency Advisory Committee on Water Data, 1982) uses the moments of the logarithms of the floods flows $X = log\ Q$.

A variation on the method of moments, which has proved effective in hydrology with the generalized extreme value distribution, is the method of probability-weighted moments or equivalently L–moments (Hosking and others, 1985; Hosking and Wallis, 1997). Probability-weighted moments or the corresponding L–moments provide a different way to summarize the statistical properties of hydrological datasets (Hosking, 1990). An advantage of L–moment estimators are that they are linear combinations of the observations and thus do not involve squaring and cubing the observations. As a result, the L-coefficient of variation and L-skewness are almost unbiased, whereas the product-moment estimators of the coefficient of variation and coefficient of skewness are highly biased and highly variable (Vogel and Fennessey, 1993). This is of particular value for regionalization procedures, which is further discussed in 5.9.

L–moments are another way to summarize the statistical properties of hydrological data based on linear combinations of the original data (Hosking, 1990). Recently, hydrologists have found that regionalization methods that use L–moments are superior to methods that use traditional moments. They have also worked well for fitting some distributions with on-site data (Hosking and others, 1985). The first L–moment is the arithmetic mean:

$$\lambda_1 = E[X] \tag{5.25}$$

Let $X_{(i|n)}$ be the i^{th} largest observation in a sample of size n (i = 1 corresponds to the largest). Then, for any distribution, the second L–moment is a description of scale based on the expected difference between two randomly selected observations:

$$\lambda_2 = (1/2)\ E[X_{(1|2)} - X_{(2|2)}] \tag{5.26}$$

Similarly, L–moment measures of skewness and kurtosis use:

$$\lambda_3 = (1/3)\ E[X_{(1|3)} - 2\ X_{(2|3)} + X_{(3|3)}] \tag{5.27}$$

$$\lambda_4 = (1/4)\ E[X_{(1|4)} - 3\ X_{(2|4)} + 3\ X_{(3|4)} - X_{(4|4)}] \tag{5.28}$$

Just as product moments can be used to define dimensionless coefficients of variation and skewness, L–moments can be used to define a dimensionless L-coefficient of variation and an L–coefficient of skewness (Table II.5.4). L–moment estimators have often been computed based on an intermediate statistics called probability-weighted moments (Hosking, 1990; Hosking and Wallis, 1997; Stedinger and others, 1993). Many early studies used probability-weighted moment estimators based on plotting positions (Hosking and others,

Table II.5.4. Dimensionless statistics used to describe distributions (product–moment and L–moment ratios)

Name	*Denotation*	*Definition*
Product–moment ratios		
Coefficient of variation	CV_X	σ_X/μ_X
Coefficient of skewness[a]	γ_X	$E[(X - \mu_X)^3] / \sigma_X^3$
Coefficient of kurtosis[b]	–	$E[(X - \mu_X)^4] / \sigma_X^4$
L–moment ratios[c]		
L–coefficient of variation	L–CV, τ_2	λ_2/λ_1
L–coefficient of skewness	L–skewness, τ_3	λ_3/λ_2
L–coefficient of kurtosis	L–kurtosis, τ_4	λ_4/λ_2

[a] Some texts define $\beta_1 = [\gamma_x]^2$ as a measure of skewness.
[b] Some texts define the kurtosis as $\{E[(X - \mu_x)^4]/\sigma_x^4 - 3\}$; others use the term excess kurtosis for this difference because the normal distribution has a kurtosis of 3.
[c] Hosking (1990) uses τ instead of τ_2 to represent the L–CV ratio.

1985); these were later found to lack the consistency and invariance required of such estimators (Hosking and Wallis, 1995; Fill and Stedinger, 1995), so that subsequent work has shifted to use the unbiased probability-weighted moment estimators. Direct estimation of unbiased L–moments from a sample is described by Wang (1996).

A method that has very strong statistical motivation is maximum likelihood. It chooses the parameters which make a fitted distribution as consistent as possible, in a statistical sense, with the observed sample. Maximum likelihood estimators are discussed in general statistics textbooks and are recommended for use with historical and paleoflood records because of their ability to make particularly efficient use of censored and categorical datasets.

Non-parametric methods can be employed to estimate the flood–flow frequency relationship, offering the advantage that one need not assume that floods are drawn from a particular parametric family of distributions. These methods have been adopted in Canada (Pilon and Harvey, 1992).

5.5.2 Use of logarithmic transformations

When data vary widely in magnitude, which frequently occurs in water quality monitoring, the sample product-moments of the logarithms of data are often employed to summarize the characteristics of a dataset or to estimate distribution parameters. A logarithmic transformation is an effective vehicle for normalizing values that vary by order of magnitude, and for preventing occasionally large values from dominating the calculation of product-moment estimators. However, the danger of using logarithmic transformations is that unusually small observations or low outliers are given greatly increased weight. This is of concern when large events are of interest and when small values are poorly measured. Small values may reflect rounding errors or may be reported as zero when they fall below a certain threshold.

5.5.3 Historical information

In addition to a relatively brief period of systematic measurements, there may be additional historical information available that pertains, for example, to the magnitude of floods prior to the commencement of the systematic collection of records. A gauging station might have only 20 years of measurement records as of 1992, yet it might be known that in 1900 a flood occurred with a peak which exceeded any flood measured and was also the greatest flood since the community was established in 1860. The magnitude of this flood and the knowledge that the other floods from 1860 to 1992 were less than the flood of 1900 should and can be used in the frequency analysis. In other instances, it may only be known that a certain number of floods from 1860 to 1972 exceeded a certain threshold. This is also historical information and should be included in the frequency analysis. Different processes generate historical and physical paleoflood records. Floods leaving a high-water mark are the largest to have occurred during the corresponding period, whereas slackwater sediment deposits in protected areas can provide evidence of the magnitude of a number of large floods.

Apart from the routine monitoring of streamflow, certain floods may be recorded simply because they exceed a perception level and have sufficiently disrupted human activities for their occurrence to have been noted, or for the resultant physical or botanical damage to be available with which to document the event (Stedinger and Baker, 1987; Wohl, 2000). Several methods can be used to incorporate historical information into the estimation of the parameters of the mathematical distribution function. They are historically adjusted weighted moments, maximum likelihood, the expected moments algorithm and the non-parametric method (Cohn and others, 2001; England and others, 2003; Griffis and others, 2004). It has been shown that the maximum likelihood method makes more efficient use of the additional information than historically weighted moments. Maximum likelihood estimators and expected moments algorithms are both very flexible and appear to be equally efficient with the LP3 distribution for which expected moments algorithms were developed, though maximum likelihood estimators often have convergence problems with those distributions.

5.5.4 Record augmentation

It is often possible to effectively extend a short record using a longer record from a nearby station with which observations in the the short record are highly correlated. In particular, a long series from a nearby station can be used to improve estimates of the mean and variance of the events that occur at the short-record site. For this purpose, it is not necessary to actually construct the extended series; one only needs the improved estimates of the moments. This idea of record augmentation is developed in Matalas and Jacobs (1964); see also the Interagency Advisory Committee on Water Data

(1982), (Appendix 7). Recent improvements and a discussion of the information gain are provided by Vogel and Stedinger (1985). In other instances, a longer series can be created that will be employed in simulation or will be archived. The idea of using record extension to ensure that generated flows have the desired mean, variance and correlations is developed by Hirsch (1982), Vogel and Stedinger (1985) and, where multivariates are concerned, by Grygier and others (1989).

5.5.5 Analysis of mixed populations

A common problem in hydrology is that annual maximum series are composed of events that may arise from distinctly different processes. For example, precipitation may correspond to different storm types in different seasons, such as summer thunderstorms, winter frontal storms, and remnants of tropical hurricanes or snowmelt. Floods arising from these different types of events may have distinctly different distributions. Waylen and Woo (1982) examined summer runoff and winter snowmelt floods separately. Vogel and Stedinger (1984) studied summer rainfall and winter ice-jam-affected floods. Hirschboeck and others (2000) considered the categorization of different floods above a specific threshold into classes based on the prevailing synoptic weather pattern; this resulted in a mixed-population flood analysis using a partial duration series framework. In some mountainous regions in small basins, summer thunderstorms produce the largest floods of record, but snowmelt events produce most of the maximum annual events. In such instances, as illustrated by Waylen and Woo (1982), separation of the flood record into separate series can result in a better estimate of the probability of extreme events because the data describing phenomena that produce those large events is better represented in the analysis.

Suppose that the annual maximum series M_t is the maximum of the maximum summer event S_t and the maximum winter event W_t:

$$M_t = max\ \{S_t,\ W_t\} \qquad (5.29)$$

Here S_t and W_t may be defined by a rigidly specified calendar period, a loosely defined climatic period, or the physical and meteorological characteristics between the phenomena that generated the observations.

If the magnitudes of the summer and winter events are statistically independent, meaning that knowing one has no effect on the conditional probability distribution of the other, the probability distribution for the annual maximum event M is given by (Stedinger and others, 1993):

$$F_M\ (m) = P[M = max(S,\ W) \leq m] = F_S(m)\ F_W\ (m) \qquad (5.30)$$

For two or more independent series of events contributing to an annual maximum, the distribution of the maximum is easily obtained. If several statistical-dependent processes contribute to an annual maximum, the distribution of the maximum is much more difficult to calculate from the distributions of the individual series. An important issue is that of deciding whether it is advisable to model several different component flood series separately, or whether it is just as reasonable to model the composite maximum annual series directly. If several series are modelled, then more parameters must be estimated, but more data are available if the annual maximum series or the partial duration series for each type of event is available.

The idea of the mixing of two distributions led to the devolopment of a two-component extreme value ß distribution by Rossi and others (1984), which corresponds to the maximum of two independent EV1 distributions. It can be thought of as the maximum of two flood processes in a partial duration series, each with Poisson arrivals and exponentially distributed flood peaks. Generally, one of the two distributions is thought of as describing the bulk of the data, and the other as the outlier distribution. Because the model has four parameters, it is very flexible (Beran and others, 1986). Therefore, if only the annual maximum series are used, regional estimation methods are essential to resolve the values of all four parameters, making regional two-component extreme value estimators an attractive option. The two-component extreme value distribution has been successfully employed as the basis of index flood procedures (Frances, 1998; Gabriele and Villani, 2002). The non-parametric distribution (Adamowski, 1985) and Wakeby distribution (Pilon and Harvey, 1992) can also be used to model the mixture distribution.

5.5.6 Frequency analysis and zeros

Low-flow series often contain years with zero values, while some sites' maximum series may also contain zero values for some sites. In some arid areas, zero flows are recorded more often than non-zero flows. Streamflows recorded as zero imply either that the stream was completely dry, or that the actual streamflow was below a recording or detection limit. This implies that some low-flow series are censored datasets. Zero values should not simply be ignored and do not necessarily reflect accurate measurements of the minimum flow in a channel. Based on the hydraulic configuration of a gauge and on knowledge of the rating curve and recording policies, it is possible to determine the lowest discharge that can be reliably estimated and would not be recorded as a zero. The plotting position method and the conditional probability model are reasonable procedures for fitting a probability distribution with datasets containing recorded zeros. The graphical plotting position approach, without a formal statistical model, is often sufficient for low-flow frequency analyses. The low-flow frequency curve can be defined visually and the parameters of a parametric distribution can be estimated by using probability-plot regression as described by Kroll and Stedinger (1996) and Stedinger and others (1993), or by using non-parametric methods.

5.6 PROBABILITY PLOTS AND GOODNESS-OF-FIT TESTS

5.6.1 Plotting positions and probability plot

Initial evaluation of the adequacy of a fitted probability distribution is best done by generating a probability plot of the observations. When the sorted observations are plotted against an appropriate probability scale, except for sampling fluctuation, they fall approximately on a straight line.

Such a plot serves both as an informative visual display of the data and a check to determine whether the fitted distribution is consistent with the data.

Such plots can be generated with special commercially available probability papers for some distributions, including the normal, two-parameter log-normal and Gumbel distributions, all of which have a fixed shape. Thanks to modern software, however, it is generally easier to generate such plots without the use of special papers (Stedinger and others, 1993). The i^{th} largest observed flood $x_{(i)}$ is plotted versus the estimated flood flow associated with the exceedance probability, or probability-plotting position q_i, assigned to each ranked flood $x_{(i)}$; $x_{(1)} > x_{(2)} > \ldots > x_{(n)}$. The exceedance probability of the i^{th} largest flood $x_{(i)}$ can be estimated by any of several reasonable formulae. Three commonly used are the Weibull formula with $p_i = i / (n + 1)$, the Cunnane formula with $p_i = (i - 0.40) / (n + 0.2)$, and the Hazen formula with $p_i = (i - 0.5) / n$. Cunnane (1978) and Adamowski (1981) provide a discussion of the plotting position issue. Plotting positions for records that contain historical information is developed in Hirsch and Stedinger (1987). Hydrologists should remember that the actual exceedance probability associated with the largest observation in a random sample has a mean of $1/(n+1)$ and a standard deviation of nearly $1/(n+1)$ (Stedinger and others, 1993); thus all of the plotting positions give only crude estimates of the relative range of exceedance probabilities that could be associated with the largest events (Hirsch and Stedinger, 1987).

5.6.2 Goodness-of-fit tests

Several rigorous statistical tests are available and are useful in hydrology to determine whether it is reasonable to conclude that a given set of observations was drawn from a particular family of distributions (Stedinger and others, 1993). The Kolmogorov–Smirnov test provides bounds within which every observation on a probability plot should lie if the sample is actually drawn from the assumed distribution (Kottegoda and Rosso, 1997). The probability plot correlation test is a more effective test of whether a sample has been drawn from a postulated distribution (Vogel and Kroll, 1989; Chowdhury and others, 1991). Recently developed L–moments can be used to assess if a proposed Gumbel-, generalized extreme value- or normal distribution is consistent with a dataset (Hosking, 1990; Chowdhury and others, 1991). Discussion of the development and interpretation of probability plots is provided by Stedinger and others, (1993) and Kottegoda and Rosso (1997).

5.6.3 Information criteria

Many approaches have been suggested for the comparison of flood distributions. Goodness-of-fit tests have been applied to assess the suitability of different probability distributions for describing annual maximum flow series, and to evaluate simulated samples in the case of simulation studies. These tests establish which distributions are, in general, the most appropriate for flood modeling. To assess the quality of a fitted model, Akaike (1974) introduced an information criterion called AIC, which stands for Akaike information criterion. It can be adapted to many different situations and consists in minimizing an information measure. The information criterion is defined as follows:

$$AIC(f) = -2log\, L(\hat{\theta}, x) + 2k \tag{5.31}$$

where $L(\hat{\theta}, x)$ is the likelihood function, and k is the number of parameters. According to Akaike (1974), the model that better explains the data with the least number of parameters is the one with the lowest Akaike information criterion. To select an appropriate model, some compromises between the goodness of fit and the complexity of the model must be accepted. Alone, the Akaike information criterion is not appropriate for model selection.

A Bayesian extension of the minimum Akaike information criterion concept is the Bayesian information criterion called BIC. It is defined as follows:

$$BIC(f) = -2log\, L(\hat{\theta}, x) + k\, log(n) \quad (5.32)$$

where $L(\hat{\theta}, x)$ the likelihood function, k is is the number of parameters and n is the sample size. The Bayesian information criterion is also a parsimony criterion. Of all the models, the one with the lowest Bayesian information criterion is considered to be best. The Schwarz method (1978) is often used to obtain the Bayesian information criterion . However, this method can also be used to get an asymptotic approximation of a Bayes factor. Furthermore, it can be combined to an a priori probability distribution to obtain the a posteriori probability for each distribution of a given set of distributions. Bayesian information criteria have not yet been used much in hydrology, however. The above-mentioned methods, which merit broader use, are available in HYFRAN software. Ozga-Zielinska and others (1999) developed a computer package for calculating design floods when a sufficiently long period of record is available. There are many other computer packages, including those listed in HOMS.

5.7 RAINFALL FREQUENCY ANALYSIS [HOMS I26, K10, K15]

The frequency of occurrence of rainfall of different magnitudes is important for various hydrological applications. In particular, rainfall frequency analyses are used extensively to plan and design engineering works that control storm runoff, such as dams, culverts, urban and agriculture drainage systems. This is because, in most cases, good-quality flow data of a length adequate for the reliable estimation of floods are generally limited or unavailable at the location of interest, while extensive precipitation records are often available. In general, there are two broad categories of approaches for estimating floods from precipitation data: those employing the statistical analysis of precipitation data and those based on the deterministic estimation of the so-called probable maximum precipitation (PMP). While it has been used worldwide for the design of various large hydraulic structures, probable maximum precipitation does not provide probability estimates for risk assessment work. The main part of this section focuses, therefore, on statistical rainfall estimation methods that can provide both flood magnitudes and associated probabilities; the second part deals with the estimation of extreme rainfall. The theory and applications of PMP have been well documented in hydrological and engineering literature such as the *Manual for Estimation of Probable Maximum Precipitation* (WMO-No. 332) and NRCC (1989); and are summarized in 5.7.5.6.

The main objective of rainfall frequency analysis is to estimate the amount of precipitation falling at a given point or over a given area for a specified duration and return period. Results of this analysis are often summarized by intensity–duration–frequency relationships for a given site or are presented in the form of a precipitation frequency atlas, which provides rainfall accumulation depths for various durations and return periods over the region of interest. For instance, estimates of rainfall frequencies for various durations, ranging from 5 minutes to 10 days, and return periods from 1 to 100 years are available. Such data can be found for the United States in the US Weather Service and Atlas series of the National Oceanic and Atmospheric Administration (Frederick and others, 1977), for Australia in *Australian Rainfall and Runoff: A Guide to Flood Estimation* (Pilgrim, 1998), for Canada in the *Rainfall Frequency Atlas for Canada* (Hogg and Carr, 1985) or in the *Handbook on the Principles of Hydrology* (Gray, 1973) and for the United Kingdom in the *Flood Estimation Handbook* (Institute of Hydrology, 1999).

Basic considerations of frequency analysis of hydrological data are discussed in 5.1 to 5.6, whereas special applications for rainfall analysis are covered in 5.7. The statistical methods described herein apply to storm or other short-duration rainfall data. Similar methods are used for flood peaks, flood volumes, low flows, droughts and other extreme events. In particular, the selection of distribution types for extremes of precipitation is discussed by WMO (1981).

5.7.1 Assessment of rainfall data for frequency analysis

Rainfall data used for frequency analysis are typically available in the form of annual maximum series, or are converted to this form using continuous records of hourly or daily rainfall data. These series contain the largest rainfall in each complete year of record. An alternative data format for precipitation frequency studies is partial duration series, also referred to as peaks over threshold data, which consist of all large precipitation amounts above certain thresholds selected for different durations. The difference in design rainfall estimates using annual maximum and partial duration series was found to be important for short return periods of two to five years but insignificant for long return periods of ten years or longer (Chow, 1964; Stedinger and others, 1993).

As for any statistical analyses, both the quantity and quality of the data used are important. The precipitation data should be collected for a long period of time. A sufficiently long record of precipitation data provides a reliable basis for frequency determinations. It is known that a data sample of size n, in the absence of a priori distributional assumptions, can furnish information only about exceedance probabilities greater than approximately 1/n (NRC, 1988). It is a common rule of thumb to restrict extrapolation of at-site quantile estimates to return periods (years) of up to twice as long as the record length (NRCC, 1989). Hence, long-term precipitation data are extremely valuable for determining statistically based rainfall estimates of reasonable reliability, especially for extreme rainfalls with high return periods, such as those greater than 100 years.

The quality of precipitation data may affect its usability and proper interpretation in flood frequency analysis studies. Precipitation measurements are subject to both random and systematic errors (Sevruk, 1985). The random error is due to irregularities of topography and microclimatical variations around the gauge site. Random errors are also caused by inadequate network density to account for the natural spatial variability of rainfall. The systematic error in point precipitation measurement is, however, believed to be the most important source of error. The largest systematic error component is considered to be the loss due to wind field deformation above the orifice of elevated precipitation gauges. Other sources of systematic error are wetting and evaporation losses of water that adheres to the funnel and measurement container, and rain splash. A broader discussion of systematic errors and their correction is contained in Volume I, 3.3.6, of this Guide.

As rainfall data are collected at fixed observation times, for example clock hours, they may not provide the true maximum amounts for the selected durations. For example, studies of thousands of station-years of rainfall data indicate that multiplying annual maximum hourly or daily rainfall amounts for a single fixed observational interval of 1 to 24 hours by 1.13 will yield values close to those to be obtained from an analysis of true maxima. Lesser adjustments are required when maximum observed amounts are determined from 2 or more fixed observational intervals as indicated in Table II.5.5 (NRCC, 1989). Thus, maximum 6- and 24-hour amounts determined from 6 and 24 consecutive fixed one-hour increments require adjustment by factors of only 1.02 and 1.01, respectively. These adjustment factors should be applied to the results of a frequency analysis of annual maximum series to account for the problem of fixed observational times (NRCC, 1989).

Table II.5.5. Adjustment factor for daily observation frequency

Number of observations/ days	1	2	3–4	5–8	9–24	> 24
Adjustment factor	1.13	1.04	1.03	1.02	1.01	1.00

For frequency analysis studies, it is necessary to check precipitation data for outliers and consistency. As noted in 5.4.5, an outlier is an observation that departs significantly from the general trend of the remaining data. Procedures for treating outliers require hydrological and mathematical judgment (Stedinger and others, 1993). In the context of regional analysis of precipitation, the outliers could provide critical information for describing the upper tail of the rainfall distribution. Hence, high outliers are considered to be historical data if sufficient information is available to indicate that these outlying observations are not due to measurement errors. Regarding data inconsistency, there are many causes. Changes in gauging instruments or station environment may cause heterogeneity in precipitation time series. Data from the gauge sites located in forest areas may not be compatible with those measured in open areas. Measurements in the valley and mountain stations and at various altitudes will not provide identical information regarding precipitation characteristics. Therefore, care must be used in applying and combining the precipitation data.

5.7.2 At-site frequency analysis of rainfall

A frequency analysis can be performed for a site for which sufficient rainfall data are available. Similar to flood frequency analysis, rainfall frequency analysis is also based on annual maximum series or partial duration series (for example, Wang, 1991; Wilks, 1993). Arguments in favor of either of these techniques are contained in the literature (NRCC, 1989; Stedinger and others, 1993). Owing to its simpler structure, the annual maximum series-based method is more popular. The partial duration analysis, however, appears to be preferable for short records, or where return periods shorter than two years are of interest. The choice of an appropriate technique should depend on the purpose of the analysis and characteristics of the available data in terms of both quantity and quality. Improved reliability of the results can be generally achieved with the use of sophisticated and comprehensive analysis methods. Virtually all hydrological estimates are subject to uncertainty. Therefore, it is often advisable to produce estimates using two or more independent methods and to perform a sensitivity analysis to gain information regarding the potential reliability of results.

Briefly, the steps below should be followed to determine the frequency distribution of annual maximum rainfall for a given site:

(a) Obtain a data sample and perform an assessment of data quality based on hydrological and statistical procedures;
(b) Select a candidate distribution model for the data and estimate the model parameters;
(c) Evaluate the adequacy of the assumed model in terms of its ability to represent the parent distribution from which the data were drawn.

The assessment of data quality is an important step in all statistical computations. The basic assumption in precipitation frequency analysis is that the data are independent and identically distributed. As mentioned above, precipitation measurements could be subject to various sources of error, inconsistency and heterogeneity. Detailed examination and verification of the raw data are needed to identify invalid data in the record caused by instrument malfunction and/or human error. Standard statistical tests are available to verify serial independence, stationarity and homogeneity of the data series (see 5.4).

There is no general agreement as to which distribution or distributions should be used for precipitation frequency analysis. A practical method for selecting an appropriate distribution is by examining the data with the use of probability plots. Probability plots, which require the use of a plotting position formula, are an effective tool to display graphically the empirical frequency distribution of the data and to assess whether the fitted distribution appears consistent with the data. There are several plotting-position formulae available in practice (see 5.6 and Nguyen and others, 1989) among which the Hazen, Weibull, and Cunnane formulas are the most popular. The differences between these three formulae are small for observations that are neither the largest nor the smallest; however, they can be significant for the largest three or four values in the data series (Stedinger and others, 1993). An alternative method for making a good choice among different distributions is based on the L–moment diagram (Stedinger and others, 1993).

Common distributions that have been applied to the analysis of annual maximum series include the Gumbel, generalized extreme value, log-normal, and log-Pearson type III distributions. Among these distributions, the generalized extreme value and its special form, the Gumbel distribution, have received dominant applications in modelling the annual maximum rainfall series. The Gumbel distribution was found, however, to underestimate the extreme precipitation amounts (Wilks, 1993). Adamowski and others, (1996) have shown that Canadian precipitation intensity data for various durations do not appear to be drawn from a Gumbel distribution. Studies using rainfall data from tropical and non-tropical climatic regions (Nguyen and others, 2002; Zalina and others, 2002) also suggest that a three-parameter distribution can provide sufficient flexibility to represent extreme precipitation data. In particular, the generalized extreme value distribution has been found to be the most convenient, since it requires a simpler method of parameter estimation and is more suitable for regional estimation of extreme rainfalls at sites with limited data or with no data (Nguyen and others, 2002). When the return periods associated with frequency-based rainfall estimates greatly exceed the length of record available, discrepancies between commonly used distributions tend to increase.

Many methods for estimating distribution parameters are available in the hydrological and statistical literature. The simplest method is the method of moments that provides parameter estimates indicating that the theoretical moments are equal to the computed sample moments. An alternative method for estimating parameters is based on the sample L–moments. These are found to be less biased than traditional moment estimators, and are thus better suited for use with small sample sizes. The L–moment method has proved effective in the estimation of the generalized extreme value distribution parameters (Stedinger and others, 1993). Another method is the method of maximum likelihood. This method provides estimators with very good statistical properties in large samples, but the estimators are often not available in closed form and thus must be computed using an iterative numerical method.

The reliability of precipitation frequency estimates depends on how well the fitted model represents the parent distribution. Several goodness-of-fit criteria can be used to test whether a selected distribution is consistent with a particular data sample (NRCC, 1989; Stedinger and others, 1993; ASCE, 1996). As mentioned above, probability plots are extremely useful in the assessment of the adequacy of fitted distributions. The assessment is performed by plotting observed rainfall data versus plotting-position estimates of exceedance probability on a specialized plotting paper. The estimated distribution is plotted on the same graph. Goodness of fit is judged by inspection. More rigorous statistical tests such as the Kolmogorov–Smirnov, probability plot correlation and L–moment tests are available, allowing quantitative judgment of goodness of fit. However, the selection of the distribution that best fits each dataset is not a recommended approach for frequency analysis (Stedinger and others, 1993; ASCE, 1996). The use of the best-fitting distribution for each data sample provides frequency estimates that are too sensitive to the sampling variations in the data and the period of record available. Current distribution selection procedures adopted by many countries are based on a combination of regionalization of some parameters and split-sample Monte-Carlo evaluations of different estimation methods to find distribution-estimation procedure combinations that give reliable quantile and risk estimates (Stedinger and others, 1993; ASCE, 1996).

5.7.3 Regional rainfall frequency analysis

Even a long record may be a relatively small sample of a climatic regime. A better measure of the regime at a station may be given by a smoothed map, which includes information from nearby stations that can influence point data, and thus broadens the sample. The degree of smoothing should be consistent with the spacing of observation stations and the sampling error of the stations. Too little smoothing tends to confound sampling error with spurious regional variation.

Rainfall frequency atlases have been produced by interpolation and smoothing of at-site frequency analysis results. Regional frequency analysis, which involves data from many sites, has been shown to reduce the uncertainties in quantile estimation of extreme events (Hosking and Wallis, 1988). Similarly to regional flood analyses, the following issues should be addressed when conducting regional precipitation analyses: the selection and verification of homogeneous regions, and regional distribution parameters. Several regional estimation methods have been suggested, among which identification of the regional probability distribution and the estimation of the index-flood procedure for use with the annual maximum series are the most popular. For example, Schaefer (1990) used the index flood methodology to conduct regional analyses of annual maximum precipitation data in Washington State. It has been shown that climatically homogeneous regions can be defined based on the mean annual precipitation. Further, it was found that the coefficients of variation and skew of annual maximum rainfalls vary systematically with the mean annual precipitation. Hence, all sites within a homogeneous region could be characterized by a specific three-parameter probability distribution, such as the generalized extreme value, having fixed values of the coefficients of variation and skew. However, the use of mean annual precipitation as an index variable may not be appropriate for other regions with different climatic or topographic conditions. For instance, the median of annual maximum rainfalls at a site was recommended as the index variable for regional estimation of extreme rainfalls in the United Kingdom of Northern Ireland and Great Britain (Institute of Hydrology, 1999). In general, one of the main difficulties in the application of this technique is related to the definition of homogeneous regions. Various methods have been proposed for determining regional homogeneity, but there is no generally accepted procedure in practice (Fernandez Mill, 1995; Nguyen and others, 2002).

Another regional rainfall frequency analysis method is the station-year method. This method attempts to enlarge the sample size by pooling records from a number of stations into a single large sample of size equal to the number of station years of record. Hence, when applying the station-year method, it is not advisable to estimate rainfall amounts for a site for return periods that are much longer than the length of record at any of the stations. However, the method may yield more reliable estimates if the stations can be considered to be meteorologically homogeneous, rather than using only the data originating from one site. Further, the effect of interstation correlation should be investigated because spatial correlation between samples tends to significantly reduce the number of station years.

Table II.5.6. World's greatest observed point rainfalls

Duration	*Depth (mm)*	*Location*	*Date*
1 min	38	Barot, Guadeloupe	26 November 1970
8 min	126	Fussen, Bavaria	25 May 1920
15 min	198	Plumb Point, Jamaica	12 May 1916
20 min	206	Curtea-de-Arges, Romania	7 July 1889
42 min	305	Holt, Misssouri, United States	22 June 1947
1 h 00 min	401	Shangdi, Nei Monggol, China	3 July 1975
2 h 10 min	483	Rockport, West Virginia, United States	18 July 1889
2 h 45 min	559	D'Hanis, Texas, United States	31 May 1935
4 h 30 min	782	Smethport, Pennsylvania, United States	18 July 1942
6 h	840	Muduocaidang, Nei Monggol, China	1 August 1977
9 h	1087	Belouve, Reunion Island	28 February 1964
10 h	1400	Muduocaidang, Nei Monggol, China	1 August 1977
18 h 30 min	1689	Belouve, Reunion Island	28–29 February 1964
24 h	1825	Foc Foc, Reunion Island	7–8 January 1966
2 days	2467	Aurere, Reunion Island	7–9 April 1958
3 days	3130	Aurere, Reunion Island	6–9 April 1958
4 days	3721	Cherrapunji, India	12–15 September 1974
5 days	4301	Commerson, Reunion Island	23–27 January 1980
6 days	4653	Commerson, Reunion Island	22–27 January 1980
7 days	5003	Commerson, Reunion Island	21–27 January 1980
8 days	5286	Commerson, Reunion Island	20–27 January 1980
9 days	5692	Commerson, Reunion Island	19–27 January 1980
10 days	6028	Commerson, Reunion Island	18–27 January 1980
11 days	6299	Commerson, Reunion Island	17–27 January 1980
12 days	6401	Commerson, Reunion Island	16–27 January 1980
13 days	6422	Commerson, Reunion Island	15–27 January 1980
14 days	6432	Commerson, Reunion Island	15–28 January 1980
15 days	6433	Commerson, Reunion Island	14–28 January 1980
31 days	9300	Cherrapunji, India	1–31 July 1861
2 months	12767	Cherrapunji, India	June–July 1861
3 months	16369	Cherrapunji, India	May–July 1861
4 months	18738	Cherrapunji, India	April–July 1861
5 months	20412	Cherrapunji, India	April–August 1861
6 months	22454	Cherrapunji, India	April–September 1861
11 months	22990	Cherrapunji, India	January–November 1861
1 year	26461	Cherrapunji, India	August 1860–July 1861
2 years	40768	Cherrapunji, India	1860–1861

Revised: 29 November 1991, US National Weather Service, US Department of the Interior Bureau of Reclamation, Australian Bureau of Meteorology

Owing to the latter and the spatial heterogeneity of climatic data, this approach is seldom used in practice.

5.7.4 Frequency analysis of area-averaged rainfall

In general, a catchment-average design rainfall is often required for design flood estimation, especially for large drainage basins. For instance, when the area of a basin exceeds about 25 km^2, rainfall observations at a single station, even if at the centre of the catchment, will usually be inadequate for the design of drainage works. All rainfall records within the catchment and its immediate surroundings must be analysed to take proper account of the spatial and temporal variation of rainfall over the basin. For areas large enough for the average rainfall depth to depart considerably from that at a point, it has been found beneficial to convert point values to areal values. Frequency values for area-averaged precipitation are generally obtained by applying an areal correction factor (areal correction factor) to point precipitation values. There are many methods of transformation point values to areal estimates, with different results being obtained in the same network according to the method applied (Nguyen and others, 1981; Arnell and others, 1984; Niemzynowicz, 1982; Institute of Hydrology, 1999). The areal correction factor estimates depend on the raingauge network density and, consequently, on the accuracy of estimating the mean precipitation over an area. Most of the procedures that are used for computing mean areal precipitation from raingauge data, such as the arithmetic average method, Thiessen polygon method and inversed distance-squared method, give comparable results for long time periods; but the differences in results among the various methods increase as the time period diminishes, as for daily rainfall. Dense networks of raingauges have been used to develop depth-area-duration correction factors (Smith, 1993; Institute of Hydrology, 1999). Areal correction factors depend on local climatological conditions and therefore, whenever possible, should be derived from local data. Validation is required if areal correction factors are to be used far from the location in which they were developed.

As procedures developed for converting point precipitation frequency values to areal values are mostly empirical, alternative methods have been proposed for directly carrying out areal precipitation frequency analyses using stochastic models of the spatial and temporal distributions of rainfall (Bras and Rodriguez-Iturbe, 1985; Smith, 1993).

5.7.5 Storm rainfall analysis for hydrological design applications

For design purposes, precipitation at a given site or over an area for a specified duration and return period is commonly used in the estimation of flood potential. The use of design precipitation to estimate floods is particularly valuable in those situations where flood records are not available or not long enough at the site of interest, or they are not homogeneous due to changes of watershed characteristics such as urbanization and channelization. Furthermore, design problems usually require information on very rare hydrological events: events with return periods much longer than 100 years. Common storm rainfall analysis techniques that can be used to address these design problems are discussed below.

5.7.5.1 Maximum observed rainfall

Some of the world's largest recorded rainfall amounts for selected durations are given in Table II.5.6. These values, which represent the current upper bounds on observed precipitation, are enveloped by the following approximate equation:

$$P = 422T^{0.475} \qquad (5.33)$$

where P is the rainfall depth in millimetres, and T is the duration in hours. Most locations in the world will never come close to receiving these extreme rainfall amounts.

5.7.5.2 Rainfall intensity or depth–duration–frequency relationships

In standard engineering practice, the results of point-rainfall frequency analysis are often summarized by intensity–duration–frequency relationships or depth–duration–frequency relationships for each raingauge site with sufficient rainfall records. These relationships are commonly available in both tabular and graphical form for rainfall intensities or depths at time intervals ranging from five minutes to two days and for return periods from two to one hundred years. Owing to the uncertainties involved in extrapolation, rainfall values are generally not provided for return periods longer than roughly twice the raingauge record. Empirical equations expressing intensity–duration–frequency and depth–duration–frequency relationships have been developed. There are many such equations appearing in the technical literature, of which the following forms are the most typical:

$$i = \frac{a}{t^c + b} \qquad (5.34)$$

$$i = \frac{aT}{t^c + b} \qquad (5.35)$$

$$i = a(t - b)^{-c} \qquad (5.36)$$

$$i = \frac{a + b \log T}{(1 + t)^c} \qquad (5.37)$$

where i is the average rainfall intensity, that is, depth per unit time, generally expressed in mm/hr, t is the rainfall duration in minutes or hours, T is the return period in years, and a, b and c are coefficients varying with the location and return period.

5.7.5.3 Temporal and spatial extrapolation of point rainfall estimates

A number of publications (NRCC, 1989; ASCE, 1996; Pilgrim, 1998; Institute of Hydrology, 1999) provide mapped regional analysis of precipitation frequencies for various return periods and durations. For instance, the US Weather Bureau provides a rainfall atlas that contains maps for the entire United States with contour lines of rainfall amounts for durations varying from 30 minutes to 24 hours and return periods from 2 to 100 years (Hershfield, 1961). In addition to this atlas, the US National Weather Service has prepared isohyetal maps for rainfall events having durations from 5 to 60 minutes and for return periods of 2, 10, and 100 years for the eastern and central states (Frederick, and others, 1977). This set of maps is useful for estimating design rainfalls of short duration or developing intensity–duration–frequency relationships.

Quantile estimates of point rainfall for durations and return periods not shown on the regional rainfall maps can be obtained by interpolation. For instance, for the eastern and central regions of the United States, depths for 10- and 30-minute durations for a given return period are computed by interpolation from the available 5-, 15- and 60-minute data for the same period (Frederick, and others, 1977):

$$P_{10min} = 0.41P_{5min} + 0.59P_{15min} \tag{5.38}$$

$$P_{30min} = 0.51P_{15min} + 0.49P_{60min} \tag{5.39}$$

For return periods other than 2 or 100 years, the following equations are used:

$$P_{Tyr} = aP_{2yr} + bP_{100yr} \tag{5.40}$$

in which *a* and *b* are empirical coefficients varying with return period values. Please note that these relationships are merely for illustration purposes. Owing to the regional variation in such a relationship, its application should be based on climatic similarity between the regions of its derivation and use.

In the absence of short-duration rainfall data, either at a site or sufficiently nearby for interpolation, it may be possible to estimate the rainfall regime from any indirect data that may be available. Such data include mean annual precipitation and mean annual number of days with rain, which may be obtained from maps or otherwise estimated. For the United States, the average relationship of precipitation per precipitation day (mean annual precipitation divided by days of precipitation with a base of one millimetre) to a 2-year 24-hour rainfall is as follows:

Precipitation per precipitation day (mm)	5	8	10	13
2-year 24-hour rainfall (mm)	36	56	79	107

Again, the relationship given in this table is merely for illustration. Owing to the regional variation in such a relationship, its application should be based on climatic similarity between the regions of its derivation and use.

For durations of less than 24 hours, it is advisable to estimate the 1-hour rainfall frequency amounts from the 24-hour values, to interpolate for intermediate durations and to extrapolate for durations one hour. The 2-year 1-hour rainfall is related to the 2-year 24-hour rainfall according to the mean annual number of days with thunderstorms. Studies that have included a wide range of climate indicate the following relationship:

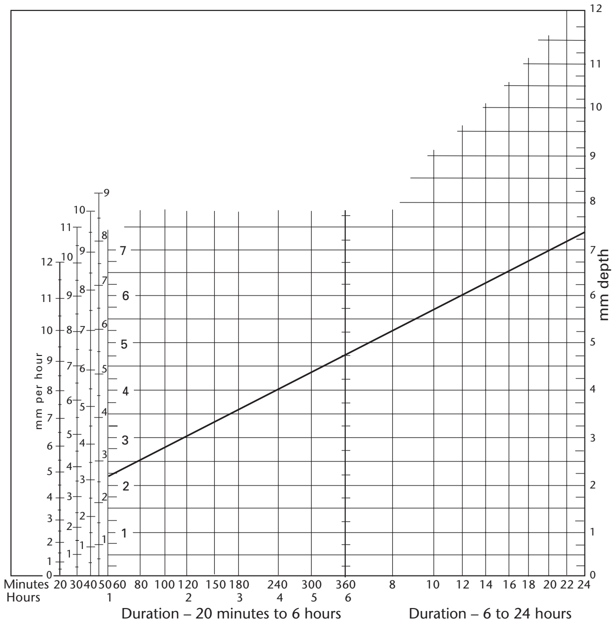
Ratio of 2-year 1-hour rainfall to 2-year 24-hour rainfall	0.2	0.3	0.4	0.5
Mean annual number of thunderstorm days	1	8	16	24

Rainfall-frequency values for durations of less than one hour are often obtained by indirect estimation. Rainfall data for such short durations are seldom readily available in convenient form for the compilation of annual or partial duration series for direct frequency analysis. Average ratios of rainfall amounts for 5, 10, 15 and 30 minutes to 1-hour amounts, computed from hundreds of station-years of records, are often used to estimate rainfall-frequency data for these short durations. These ratios, which have an average error of less than 10 per cent, are as follows:

Duration (minutes)	5	10	15	30
Ratio (n minutes to 60 minutes)	0.29	0.45	0.57	0.79

Thus, for example, if the 10-year 1-hour rainfall is 70 mm, the 10-year 15-minute rainfall is 57 per cent of 70, or 40 mm.

These ratios can yield erroneous results in some regions. For example, in regions where most of the rainfall occurs in connection with thunderstorms, the above ratios would tend to yield values that are too low. However, in regions where most of the rainfall results from orographic influences with little severe convective activity, the ratios might tend to yield values that are too high. This variation has been handled on a continental basis for Australia (Court, 1961; Hershfield, 1965), with a relationship that was developed by using a geographical location and 1-hour rainfall intensity as variables. The relationship is also dependant upon the average recurrence interval. When large quantities of rainfall data for a region are to be subjected to frequency analysis, as is usual in the preparation of generalized maps, the compilation of annual series data for



Note: For 20-minute to 60-minute rainfalls, values are in mm per hour; for longer durations the values are in mm of depth.

Figure II.5.2. Rainfall–intensity and depth–duration relationship

all durations is a challenging and tedious task. It is customary, therefore, to limit such compilations to data from a relatively small number of recording stations with good records for at least ten years. The means of the annual series are then computed and used to prepare a diagram such as that given in Figure II.5.1, which permits the estimation of rainfall values for durations of up to 24 hours when the 1- and 24-hour amounts are known. The diagonal line in Figure II.5.2 illustrates an example in which 24-hour rainfall is about 73 mm and 1-hour rainfall is 22 mm. Values for other durations can be read off the intersections of the diagonal. Thus, the amount for 12 hours is 60 mm; for two hours it is 30 mm.

Diagrams similar to Figure II.5.3 may be constructed (Miller and others, 1973) for interpolating between the 2- and l00-year return periods. Such diagrams must be based on good long-record stations if they are to be reliable. As with the duration–interpolation diagrams, they vary from region to region, where climatic regimes differ significantly. They are used in the same manner as the duration–interpolation diagrams in that a diagonal is laid across the appropriate 2- and l00-year rainfall depths on their respective verticals, and depths for other return periods are read at the intersections of the diagonal with the corresponding verticals.

With the use of the above two types of interpolation diagrams, only the 1- and 24-hour rainfall amounts for the 2- and l00-year return periods need be computed for most of the stations in the region for which the diagrams were developed. The diagrams are then used to estimate other required values. Both types are subject to regional variations, and caution should be exercised in trying to apply the diagrams in regions other than those for which they were developed.

Another method for estimating extreme rainfall quantiles for locations without rainfall data is based on regional maps of rainfall statistics. For example, Environment Canada provides maps showing isolines of the mean and the standard deviation of annual rainfall extremes for each region of Canada for durations varying from 5 minutes to 24 hours (NRCC, 1989). Hence, if the Gumbel distribution is assumed to be acceptable for describing rainfall extreme distribution, the quantile estimate of rainfall for a given return period at an ungauged location can be computed using the frequency factor method and the corresponding interpolated values of rainfall statistics. Similarly, for Australia, under the assumption of log-normal and log-Pearson type III distributions for rainfall extremes, maps of regionalized skewness along with available rainfall frequency maps can be employed to derive intensity–duration–frequency curves for any given location using appropriate extrapolation and interpolation procedures (Pilgrim, 1998).

In summary, one of the main challenges for engineers and hydrologists is to obtain representative

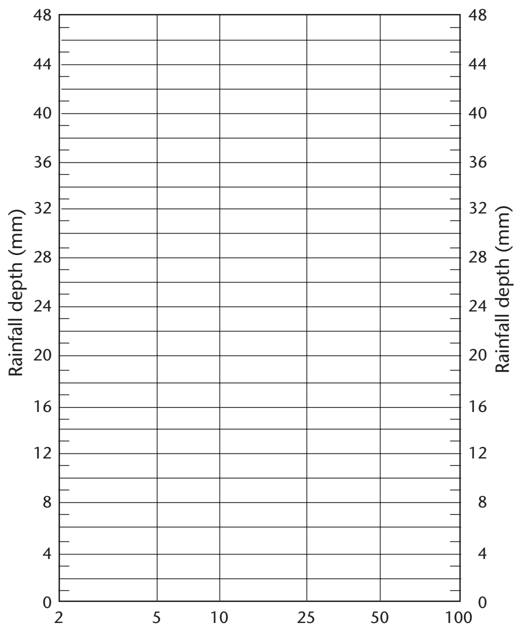


Figure II.5.3. Return-period interpolation diagram

information related to rainfall extremes at a given site. Precipitation stations, however, are not typically within close proximity to the site of interest, or they do not contain a sufficient period of rainfall records to allow a reliable estimation of rainfall. The rainfall frequency maps should be examined since they are sometimes based on the analysis of limited data over rather restricted areas, and the interpolation of rainfall characteristics to other areas could lead to grave uncertainties. Appropriate regional rainfall analysis procedures described in 5.7.3 should be used, especially for ungauged locations and for sites with limited rainfall records.

5.7.5.4 Mass rainfall curves

The first step in a storm-rainfall study is to plot accumulated values of rainfall versus time of day to give a mass curve, or integrated curve, for each station or for selected representative stations, if there are many. The mass curves for non-recording stations are constructed by comparison with mass curves from recording stations by means of proportionality factors. In doing so, the movement of the storm and any reports of the times of beginning, ending and heaviest rainfall should be taken into account. Figure II.5.4 shows a typical set of mass curves from the storm of 31 March–2 April 1962 in south-eastern Canada.

The pertinent stations are then listed in a table and accumulated values of rainfall are tabulated for each station for a pre-selected time increment. A 6-hour time increment is used in the present example, but other increments may serve equally well. For convenience, the stations should be listed in order of decreasing magnitude of total storm rainfall. The next step is to examine the table and select the particular 6-hour period that has the largest 6-hour rainfall increments. The values for this time increment are then listed. The period of maximum

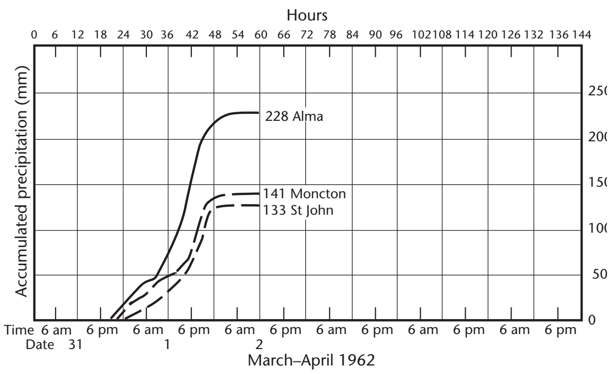


Figure II.5.4. Mass rainfall curves

Table II.5.7. Maximum average rainfall depth (mm) – storm of 31 March to 2 April 1962, south-eastern Canada

	Duration (hours)				
Area (km²)	*6*	*12*	*18*	*24*	*42*
25	90	165	205	230	240
100	85	155	190	215	225
1 000	70	130	165	185	190
10 000	50	90	115	140	145
100 000	25	45	65	75	85

12-hour rainfall is found in a similar way and its rainfall is listed. The same operation is applied to define the maximum 18-, 24-,..., n-hour increments. For periods embracing several 6-hour increments, a considerable number of trials may be required to find the period that includes the maximum rainfall for a particular duration.

5.7.5.5 Depth–area–duration analysis

Storm-rainfall analysis expresses the depth–area–duration characteristics of the rainfall from a particular storm. The depth is defined for pertinent combinations of enveloping area and duration, and is generally portrayed by tables or curves. In the aggregate, such analyses provide useful records for the design of flood control structures and for research in quantitative precipitation forecasting.

Individual point-rainfall observations are analysed jointly and in combination with other information. The rainfall data usually consist of observations of daily totals, interspersed with a few recorder measurements that contain short-term rainfall intensity information. Sometimes, these data are augmented by observations obtained through special interviews, referred to as bucket surveys. Additional information may come from synoptic weather maps, radar, reports of rises of small streams and other sources. The procedure, which is summarized in the following subsections, is described in detail in the *Manual for Depth–Area–Duration Analysis of Storm Precipitation* (WMO-No. 237).

Based on the tabulation of maximum rainfall increments, isohyetal maps are prepared for each duration, for example, 6 or 12 hours. Areas enclosed by each isohyet are then evaluated by using a planimeter or by tallying grid points, and the resulting values are plotted on a graph of area versus depth, with a smooth curve drawn for each

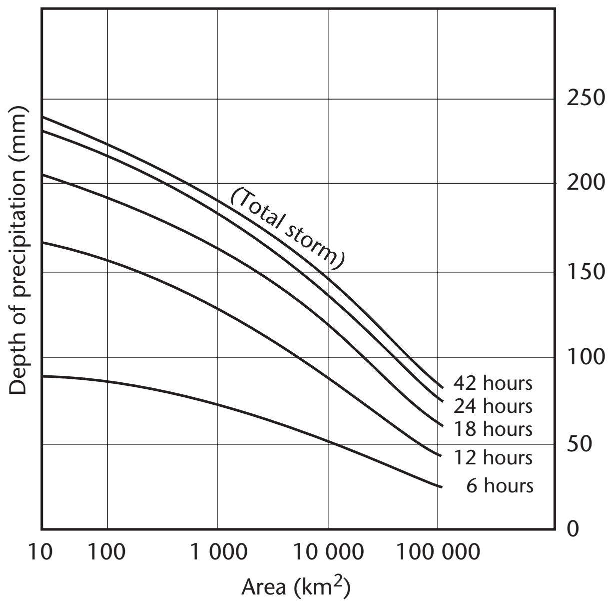


Figure II.5.5. Enveloping depth–area–duration curves

duration. A linear scale is commonly used for depth and a logarithmic scale for area. The enveloping or maximum depth–area–duration data for each increment of area and duration may be tabulated as in Table II.5.7 from curves such as those in Figure II.5.5.

5.7.5.6 Probable maximum precipitation

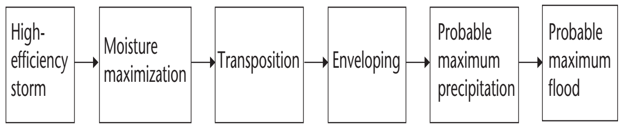
The term probable maximum precipitation, or PMP, is well established and is widely used to refer to the quantity of precipitation that approaches the physical upper limit of precipitation of a given duration over a particular basin. The terms maximum possible precipitation and extreme rainfall have been used with roughly the same meaning. To ask how possible or how probable such precipitation is would be at best a rhetorical question because the definition of probable maximum is an operational one that is specified by the operations performed on the data.

5.7.5.6.1 *Basic methods of estimating probable maximum precipitation*

There are two methods for estimating probable maximum precipitation: indirect and direct.

5.7.5.6.2 *Indirect type*

The indirect type first estimates probable maximum precipitation for the storm area, an area surrounded by isohyets, and then converts it into probable maximum precipitation for the design watershed. The main steps can be illustrated as follows:



High-efficiency storms are those for which the data support the assumption that their precipitation efficiency was near a maximum. The return period of such storms, given by point data on the enveloping curve, is usually more than 100 years.

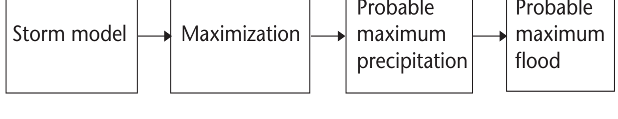
Moisture maximization is a procedure by which the moisture of a high efficiency storm is maximized. The increase is usually limited to 20–40 per cent because there is an approximate physical upper limit for the representative dewpoint, which is a critical factor, and this cannot exceed the highest water temperature of the sea surface at the source of warm and wet air masses. In addition, this decreases as one moves from the source of air masses to the design watershed.

Transposition is a procedure which accounts for moving a high-efficiency storm from one location to another within a meteorologically homogeneous zone. In essence, it replaces time with space in order to increase the number of storm samples and provide additional observed data.

Enveloping refers to the use of a depth–area–duration relationship drawn up on the basis of transposed storms, thereby maximizing precipitation depths of various area sizes and durations. This also compensates for a lack of observed data.

5.7.5.6.3 *Direct type*

The direct type estimates probable maximum precipitation for the area directly encompassing the particular project in the design watershed. Major steps include the following:



The storm model for a typical storm or for an ideal storm reflects the characteristics of the catastrophic precipitation over the design watershed which is likely to pose the greatest threat of flooding for the project. Such models can be classified as local, transposition, combination or inferential, depending on their source.

Local models are used for local storm maximization and are selected from storm data observed in the design watershed. They can also be developed by simulating historically extraordinary floods from surveys.

Transposition models are derived by transposing actual storms in surrounding similar regions.

Combination models are sequences of two or more storms that are subject to spatial or temporal storm maximization and are combined in accordance with theories of synoptic meteorology.

Inferential models are theoretical or physical models which result from generalization and inference, using the three-dimensional spatial structure of storm weather systems within the design watershed whereby major physical factors that affect precipitation are expressed by a series of physical equations. They mainly include convergence models and laminar models of the flow field or wind field.

Maximization maximizes storm performance. When the storm model is that of a high-efficiency storm, then only moisture maximization is performed; otherwise both the moisture and power factors are maximized.

The above four methods are applicable to both hilly regions and plains. The fourth method is in general applicable to area of under 4 000 km^2 and durations shorter than 24 hours, whereas the other three methods are independent of area size and duration, and work especially well for estimating probable maximum precipitation for large watersheds larger than 50 000 km^2 and durations greater than three days.

Probable maximum precipitation can also be estimated by using the statistical estimation method and the empirical formula method.

5.7.5.6.4 *Preliminary considerations*

For major structures, the cost of the spillway may be a substantial proportion of the total project cost. Its design is therefore important enough to warrant a very detailed study. However, in the preliminary planning stages, it is sufficient to use generalized estimates of probable maximum precipitation if these are available for the area. Estimates of this type for the United States have been published as maps and diagrams in various issues of the US Weather Bureau Hydrometeorological Report series. Similar reports have been prepared by several other countries for various parts of the world. The following steps should be taken when determining probable maximum precipitation:

(a) Value basic data. Collect necessary hydrometeorological, geographic and orographic data, especially those related to extraordinary storms or floods and corresponding meteorological data, and asses their reliability;
(b) Make full use of storm data. Such data for the design watershed and its surrounding regions are the basis for calculating probable maximum precipitation and are also one of the major factors influencing the precision of results;
(c) Analyse characteristics and causes of large storms in the design watershed in order to provide a basis for determining methods for calculating probable maximum precipitation, selecting indicators, maximizing and analysing the reasonableness of results;
(d) Fully understand the characteristics of the methods. Select two or more methods from those that are available for determining probable maximum precipitation based on the conditions required for each method and the design requirements and data available for the watershed. Complete the calculation separately and then select the final results by means of a comprehensive evaluation.

5.7.5.6.5 *Requirements for probable maximum precipitation*

Unless depth–area–duration analyses applied to a project basin have been constructed within the storm-transposition zone, a number of individual storm studies will be required to obtain estimates of probable maximum rainfall. Before these studies are undertaken, the likely critical rainfall duration for the particular design problem should be determined. The selection of an appropriate tentative rainfall duration design can help avoid the analysis of data that are not directly applicable to the project and the subsequent need for analysis of additional data if too short a duration is adopted in the first instance.

The approximate time of rise of flood hydrographs for storms centring on different parts of the basin and the particular characteristics and proposed method of operation of the projected works should be considered in selecting tentative design rainfall duration.

The calculation undertaken should depend on the storm characteristics and design requirements of the project (Ministry of Water Resources and

Ministry of Energy of the People's Republic of China, 1995):

(a) If a project design requirement calls for probable maximum precipitation of a particular duration, only the storm volume and the most severe spatial or temporal distributions of that duration need be calculated;
(b) If the project calls for probable maximum precipitation of several durations, probable maximum precipitation should be determined for each of those durations.
(c) If the project involves a series of reaches along a river, as in a cascade of dams, then a series of probable maximum precipitation estimates will need to be made, with attention being paid to coordination between the upper and lower reaches. Regional estimates of probable maximum precipitation should be in accordance with the characteristics of observed storms;
(d) For places where storm characteristics differ among seasons, probable maximum precipitation estimates should be made for summer, autumn, rainy seasons, typhoons and so forth.

5.7.5.6.6 *Selection of sub-basins*

For project sites with large drainage areas, it may be necessary to estimate the probable maximum rainfall for some sub-basins and then compound the resultant probable maximum flood hydrographs from these sub-basins. To avoid subsequent unnecessary or incomplete analyses of mean areal rainfall depths during the storm studies, the sub-basins for which flood hydrographs are required should be selected before storm analyses are started. The selection of sub-basins is influenced by the physical characteristics of the basin and the availability and locations of stream-gauging stations from which the sub-area flood hydrographs can be routed to the project site.

Three commonly used methods are summarized below: the storm transposition method, the generalized estimation method and the statistical estimation method.

5.7.5.6.7 *Storm transposition method*

The basic assumption of storm transposition is that the region where the storm occurred – the storm source – and the design region are similar in terms of geographic or orographic conditions and the synoptic causes of storms. As a result, the structure – temperature, air pressure, wind power and spatial or temporal distributions – of a transposed storm is expected to change little. It includes the two following assumptions:

(a) After transposition, the storm weather system and the relative position of the storm area change little;
(b) After transposition, spatial or temporal distributions – the hyetograph and the isohyets – of the storm also change little.

5.7.5.6.8 *Selection of transposed objects*

Analyses should first be performed on the basis of data on observed catastrophic intense rainfall or floods which were collected from the design watershed in order to understand the basic types of catastrophic rainfall or floods in the watershed and then identify the storm types corresponding to probable maximum flood, PMF, required by the design project. For example, if the event in question is a tropical cyclone (typhoon, hurricane) or a frontal storm, the transposed object should be selected from among tropical cyclone storms or frontal storms, respectively.

5.7.5.6.9 *Possibility of transposition*

This involves a study of whether the selected transposed object is likely to occur in the design watershed. There are three solutions:

(a) Identifying meteorologically homogenous zones;
(b) Setting transposition limits for a particular storm;
(c) Performing specific analyses on the design watershed and comparing the similarity between the design watershed and the region of the storm source in terms of climate, weather, geography, orography and the like. The more similar these are, the more possible the transposition.

5.7.5.6.10 *Isohyetal map allocation*

Isohyetal map allocation moves the isohyetal map of the transposed object to the design watershed, which raises questions such as where to put the storm centre, whether to rotate the direction of the storm axis – the direction of the major axis of the isohyetal map – and how to rotate it.

The computations start with a study of the statistics of the spatial distribution of actual storms, that is, finding common rules of central positions and directions of axes of storms with weather causes similar to those of the transposed object on the basis of existing storm data, including those observed, surveyed and recorded in the literature, and then making adjustments and decisions in relation to the particular circumstances of the project.

The transposed isohyets should suit the large-scale orography of the design watershed as well as possible. The storm centre should match the small-scale orography, such as that surrounding the river channel.

5.7.5.6.11 *Transposition correction*

The purpose of transposition correction is to estimate quantitative changes to rainfall caused by differences in conditions such as geometry, geography and orography of the region. In other words, transposition correction typically includes the geometric, geographic and orographic corrections of the watershed. The geographic correction considers moisture correction only, while the orographic correction includes moisture correction and power correction. The geometric correction of the watershed must be performed first for any storm transposition.

If the transposed object is very similar to the design watershed with regard to the weather situation, orographic and geographic conditions are almost the same and there is no obvious moisture obstacle in between, the storm isolines of the transposed object may be moved to the design watershed without any change. Only a geometric correction of the watershed is needed.

If the two places are similar in terms of storm weather situation and different in terms of orographic and geographic conditions, and such differences are not large enough to cause great changes to the storm mechanism, then power correction need not be considered. In this case, only moisture correction needs to be considered in addition to the geometric correction of the watershed. This method is commonly used in plains and regions of low relief.

If storms with different orographic conditions must be transposed because of actual conditions, mountains will have some effects on the storm mechanism. In such cases, power correction needs to be considered in addition to geometric and moisture corrections of the watershed.

Concern for the orientation of precipitation patterns relative to basin orientations has resulted in special studies (WMO, 1986*a*; Hansen and others, 1982).

5.7.5.6.12 *Storm maximization*

In storm transposition, selected transposed objects are typically high-efficiency storms; therefore, only moisture maximization is needed when maximizing them. For such cases, maximization may be performed at the storm source before transposition only. Only after transposition correction, is the storm probable maximum precipitation.

Maximization methods developed in the United States and adopted in a number of countries (Pilgrim, 1998) have been described by Weisner (1970) and in a number of publications of the US National Weather Service, formerly the US Weather Bureau (see references in the *Manual for Estimation of Probable Maximum Precipitation* (WMO-No. 332, 1986*a*).

5.7.5.6.13 *Generalized estimation method*

This method involves estimating probable maximum precipitation for non-orographic regions and orographic regions respectively. It is assumed that precipitation in non-orographic regions results from the passing of weather systems, while that in orographic regions results from both the passing of weather systems and orographic effects. Precipitation caused by weather systems is referred to as convergence rain, or convergence components, and those caused by orography are called orographic rains, or orographic components.

Precipitation generalization involves the generalization of convergence rains, using the depth–area–duration generalization of storms. This generalization method is applicable to both a particular watershed and a large region that includes a lot of watersheds of various sizes. For the latter, it is called generalized or regional estimation. The content of generalization includes generalization of the depth–area–duration relationship and the generalization of the spatial/temporal distributions of probable maximum precipitation.

Determining probable maximum precipitation using the depth–area–duration generalized estimation method includes four steps:

(a) Maximize actual large storms – only moisture maximization being performed in most cases;
(b) Transpose maximized storms to the study region;
(c) Smoothen and fit envelope curves to data, including depth-duration smoothing, depth-area smoothing and combined depth–area–duration smoothing;
(d) Apply the probable maximum rainfall on the storm area to the design watershed so as to determine the probable maximum storm on the watershed area.

For regional generalized estimation, regional smoothing should be added after step (c). A check for regional consistency involving numerous comparisons has been described by Hansen and others (1977) and in the *Manual for Estimation of Probable Maximum Precipitation* (WMO-No. 332).

The method is used to estimate probable maximum precipitation for durations of 6 to 72 hours and for areas under 52 000 km^2 in plains and areas under 13 000 km^2 in orographic regions in the United States. For orographic regions, the influence of the topography should be considered in probable maximum precipitation estimation. For other countries or regions, the area sizes to which the method is applicable need to be analysed, based on the actual local conditions.

The method makes full use of maxima, including the largest rainfalls for various durations and areas of all the storm data in the particular region. The results of these calculations can be coordinated in the region and the watershed.

Now widely used in the United States, Australia, India and other countries, the generalized estimation method is described in the *Manual for Estimation of Probable Maximum Precipitation* (WMO-No. 332).

Major results of the generalized estimation method include the following:
(a) The precipitation depth of probable maximum precipitation: one is the enveloping curve map of the depth–area–duration relationship of probable maximum precipitation and the other is the probable maximum precipitation isoline map for several durations and area sizes;
(b) The spatial distribution of probable maximum precipitation: generalized as a set of concentric, similar ellipses;
(c) The temporal distribution of probable maximum precipitation: generalized as a single peak;
(d) For orographic regions, there are also some correlograms or isoline maps of some parameters that reflect orographic effects, which are used to convert probable maximum precipitation of convergence rains into probable maximum precipitation for orographic regions.

5.7.5.6.14 ***Statistical estimation method***

This is an approximate method for estimating probable maximum precipitation for small watersheds, usually those under 1 000 km^2. It is summarized below.

In principle, probable maximum precipitation for small watersheds may be determined using the storm transposition method. Nonetheless, when the design region lacks the moisture and wind data needed for maximization, it will be very hard to use the traditional storm transposition method. If an abstracted statistical value K_m is transposed instead of transposing directly the rainfall of a storm, the issue will be much simpler. K_m may be defined by:

$$K_m = \frac{X_m - \overline{X}_{n-1}}{S_{n-1}} \tag{5.41}$$

where X_m is the first item in the ranked observed series, that is, the very large value,

$\overline{X}_{n-1}$ is the average excluding the very large value, that is:

$$\overline{X}_{n-1} = \frac{1}{n-1}\sum_{i=2}^{n} X_i \tag{5.42}$$

S_{n-1} is the standard deviation excluding the very large value, that is:

$$S_{n-1} = \sqrt{\frac{1}{n-2}\sum_{i=2}^{n}(X_i - \overline{X}_{i-1})} \tag{5.43}$$

Clearly, the more data that are used and the more regions that are studied, then the enveloping value of K_m will be closer to the value corresponding to probable maximum precipitation.

Hershfield (1965) collected data from more than 2 600 rainfall stations, about 90 per cent of which were in the United States and developed a graphical relationship between enveloping values and means of annual series of K_m for different durations (Figure II.5.6) for the use of hydrologists.

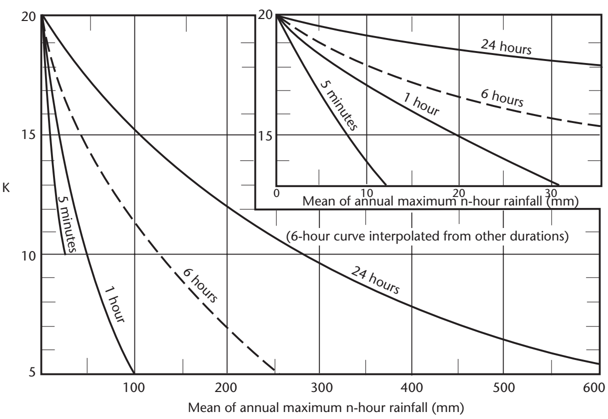


Figure II.5.6. K as a function of rainfall duration and mean of annual series

When using Figure II.5.6 to determine K_m, the average $\bar{X}_n$ and S_n are worked out based on rainfall data from a particular station in the design watershed and the calculation is completed according to the following equation:

$$PMP = \bar{X}_n + K_m S_n \quad (5.44)$$

The coefficient of variability is:

$$C_{vn} = \frac{S_n}{X_n} \quad (5.45)$$

Therefore, equation 5.39 can be rewritten as follows:

$$PMP = (1 + K_m C_{vn})\, \bar{X}_n \quad (5.46)$$

As illustrated by equation 5.46, determining probable maximum precipitation with Hershfield's statistical estimation method is essentially a matter of transposing the statistical value K_m of a very large storm in a wide region and correcting it by using the storm average $\bar{X}_n$ and the coefficient of variability C_{vn} for the design watershed. The method requires that enough single-station, daily precipitation observation series be available for the design watershed.

Maximum rainfalls needed are selected from among records using a particular duration or durations (1 hour, 6 hours, 24 hours) each year and are organized into an annual series. The mean $\bar{X}$ and the standard deviation S_n or the coefficient C_{vn} of the series are then calculated. The K value is determined from Figure II.5.6 using the mean of the series. As a

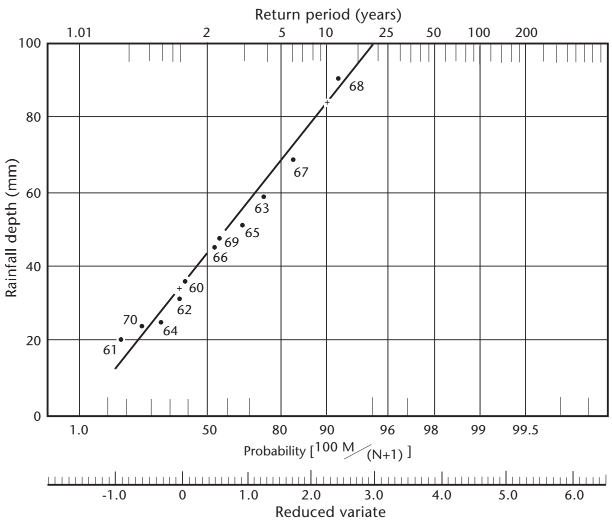


Figure II.5.7. Example of an extreme probability plot

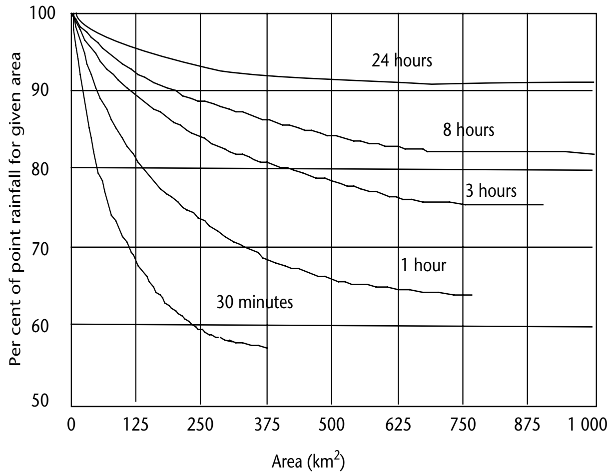


Figure II.5.8. Depth–area curves

result, probable maximum precipitation can be determined according to equation 5.44 or 5.46.

Care should be taken to ensure that the highest one or two values in the annual series are consistent with the other values comprising the series. If, for example, the maximum value in a 30-year period is twice the second-highest value, it is clearly an outlier. The easiest way to detect an outlier is to arrange the series in descending order and then compute the return period of each value. Next, the values are plotted against their corresponding return periods on probability paper as shown in Figure II.5.7. If the maximum value of the series lies well above the line delineated by the other items in the series, it can be considered an outlier. An outlier should not be used to compute the mean or standard deviation of the series. If used, the mean and standard deviation should be adjusted as indicated by Hershfield, who also provided an adjustment for length of record. A complete, detailed description of the entire procedure, including diagrams for making the necessary adjustments, is given in the *Manual for Estimation of Probable Maximum Precipitation* (WMO-No. 332), Chapter 4.

When the probable maximum precipitation is to be applied to an area larger than about 25 km², it should be reduced. No modification is considered necessary for smaller areas. For larger areas, the point value is generally reduced by means of depth area or area reduction curves similar to those of Figure II.5.8.

The statistical method described above may overestimate the probable maximum precipitation in regions of heavy rainfall and in regions of frequent storms of similar types. In regions of low rainfall and where heavy rain-producing storms, such as tropical cyclones, are rare but possible, the method may underestimate probable maximum precipitation. Values of K as high as 30 have been found necessary in order to exceed maximum observed point rainfall amounts in some regions. In some countries, in particular the United States, where storm studies are the preferred source of data for probable maximum precipitation determination, the statistical method has been used primarily as a means of checking for consistency.

5.7.5.6.15 *Checking the plausability of probable maximum precipitation estimates*

In principle, a variety of methods should be used concurrently to estimate probable maximum precipitation. Results of those methods should then be analysed comprehensively to select the best probable maximum precipitation value. In the end, the plausibility of the selected probable maximum precipitation should be checked from multiple perspectives so that the result is both maximal and possible. In general terms, methods of checking the rationality of probable maximum precipitation results are the same as those for the plausibility of probable maximum flood results. As a result, methods for checking them are the same (see 5.10.2 or *Manual for Estimation of Probable Maximum Precipitation* (WMO-No. 332), Chapter 4.

5.7.5.7 Design storm

A design storm or design hyetograph is a rainfall temporal pattern that is defined for used in the design of a hydraulic structure. A design hyetograph or synthetic storm of specified exceedance probability can be developed in the following way. The rainfall depth is obtained from the depth–duration–frequency relationship based on the specified probability and duration. Next, an area adjustment factor is applied to the rainfall depth. Finally, a method is used to distribute the rainfall depth over time using available procedures (Wenzel, 1982; Arnell and others, 1984). Pilgrim and Cordery (1975) warn that approaches overly smoothing the temporal patterns of rainfall are unsuited for design applications because the time variability of rainfall intensity often has a significant effect on the design hydrograph. Two important points noted by Pilgrim and Cordery (1975) and Huff and Changnon (1964) are that the variability of intensities diminishes with decreasing exceedance probability and the majority of extreme storms have multiple peaks of high rainfall intensity. Depth–duration–frequency relationships can be regionalized using procedures described above.

5.7.5.8 Drought

Drought is the low hydrological extreme resulting from perturbations in the hydrologic cycle over a sufficiently long time to result in a significant water deficit. Local water resources become insufficient to support the established or normal activities of the area. Droughts are interpreted and categorized broadly as meteorological, hydrological or agricultural. The meteorologist is concerned with drought in the context of a period of below-normal precipitation. To a hydrologist, drought refers to below-average flow in streams or content in reservoirs, lakes, tanks, aquifers and soil moisture. To an agriculturist, drought means a prolonged shortage of soil moisture in the root zone.

For meteorological drought, a useful means of analysis is based on the magnitude-span frequency. A simple type of analysis would compare rainfall totals for calendar months or pertinent seasons with their corresponding normal values and assess severity of drought based on negative departures from normal values. To take into account the effect of time distribution of rainfall, an antecedent-precipitation index may be used instead of total rainfall. Another way to account for the month-to-month carry-over effects of rainfall for evaluating severity of meteorological drought is the Herbst technique (Herbst and others, 1966).

The severity of agricultural drought may be judged by the drought index, a means of summarizing and periodically disseminating drought information and crop-moisture conditions on a regional basis. It can be used for evaluating the drought hazard over a sizeable area or for periodic assessment of the current extent and severity over a region.

Hydrological drought severity is related to the severity of departure from the norm of low flows and soil moisture in conjunction with excessive lowering of groundwater levels. In view of the considerable time lag between departures of precipitation and the point at which these deficiencies become evident in surface water and groundwater, hydrological drought is even further removed from the precipitation deficiency since it is normally defined by the departure of surface and subsurface water supplies from some average condition at various points in time.

5.7.5.9 Recent precipitation frequency analysis techniques

The density of raingauges has been a significant limitation in the development of precipitation frequency analysis procedures. Radar provides a potentially important source of precipitation data for frequency analyses. The most important advantage of radar for precipitation measurement is the coverage radar provides of a large area with good spatial and temporal resolutions, as small as 1 km^2 and 5 minutes. With an effective range of 200 km, a single radar can cover an area of more than 10 000 km^2.

Cluckie and others (1987) report a depth–area–duration analysis of extreme events using hourly radar rainfall totals for five-km grid squares. The need to first correct and calibrate radar data is emphasized. Depth–area–duration analysis is performed on individual storms as a means of classifying their flood producing potential. Furthermore, Cluckie and Pessoa (1990) have used radar data from northwest England to characterize actual storms, which have then been maximized and transposed to obtain probable maximum precipitation estimates for catchments areas of interest (see 5.7.5.6 for a discussion of probable maximum precipitation). Such an approach capitalizes on radar's ability to delineate storms in space and time. In addition, a program called RADMAX implements the procedure and incorporates visualization of the storm transposition step (Moore, 1993). Collier (1993) suggested the use of radar, and satellite data for cruder estimates to support probable maximum precipitation estimation by using the storm model approach.

Design problems generally require information on very rare hydrological events, namely those with return periods much longer than 100 years. Traditional techniques for addressing these design problems are mostly based on the use of probable maximum precipitation. New frequency analysis procedures, which exploit some of the tools of probable maximum precipitation, have been developed for assessing rainfall magnitudes with very long return periods. In particular, the National Research Council (1988) recommended the stochastic storm transposition techniques. In the probable maximum precipitation application, storm transposition is based on the assumption that there exist meteorologically homogeneous regions such that a major storm occurring somewhere in the region could occur anywhere else in the region, with the provision that there may be differences in the averaged depth of rainfall based on differences in moisture potential. In the stochastic storm transposition method, the frequency of occurrence of storms in the transposition region provides the link for obtaining frequency estimates of extreme storm magnitudes. The stochastic storm transposition provides estimates of the annual exceedance probability of the average storm depth over the catchment of interest. The estimate is based on regionalized storm characteristics such as maximum storm centre depth, storm shape parameters, storm orientation, storm depth and spatial variability, and on an estimation of the joint probability distribution of storm characteristics and storm occurrence within a region. An advantage of the stochastic storm transposition method is that it explicitly considers the morphology of the storms, including the spatial distribution of storm depth and its relation to the size and shape of the catchment of interest (NRC, 1988).

5.8 LOW-FLOW ANALYSES
[HOMS I80, K10]

5.8.1 General

Information on the characteristics of low flows for streams and rivers is important for planning, design and operation of water-related projects and water resource systems. Such information is used in designing wastewater treatment and storage facilities to ensure that releases do not exceed the assimilative capacity of receiving waterways, reservoir storage design for multi-purpose systems and the allocation of water for various purposes such as industrial, agricultural, domestic and in-stream ecological needs.

Low-flow frequency analysis and flow-duration curves are the two most commonly used analytical tools to help assess the low-flow characteristics of streams, and these will be described in more detail in this section. Both approaches typically require at-site continuous streamflow data for analysis, unless regional approaches are used to estimate at-site characteristics. Other characteristics that are sometimes useful include the amount of time or frequency for which flows might be below a certain threshold during a season and the volume of water or deficit that might arise during the period in which flows are below a threshold. Statistical approaches can also be used to assess these aspects. Other approaches, such as passing historical sequences of data or synthetically generated sequences through a model of the river or reservoir system can provide additional, valuable detailed information for design purposes. The latter approaches will not be covered in this Guide.

Low flows are usually sustained by depletion of groundwater reserves or by surface discharge from upstream bodies of water including lakes, wetlands and glaciers. Low flows within a year or season may result from different mechanisms forcing the hydrological response. Low flows in cold, northern climates may occur due to the prolonged winter period where precipitation occurring during this period is primarily in the form of snow, resulting in ever-decreasing flows until the occurrence of the spring freshet. A second period that produces low flows occurs during the warm season where there may be periods of significant evaporation and little precipitation. Depending on local climatology and physiography, some basins may produce low flows resulting predominately from one process or a combination of processes as described above (Waylen and Woo, 1987). It is important to understand the processes producing the low flows, as these may determine the analytical approaches taken to analyse their characteristics and results.

Anthropogenic intervention can greatly alter the natural low-flow regime. For example, increased extraction from surface water for irrigation may occur during periods of prolonged absence of rainfall, resulting in artificially suppressed flow values, compared with what naturally would have occurred. Significant extraction of groundwater for agricultural, industrial and human uses can reduce water-table levels and result in reduced streamflow. A variety of other anthropogenic interventions can occur within a basin and should be known prior to proceeding with analyses of data. Such interventions can include upstream regulation, inter-basin transfers, return flows from domestic sewage systems that use groundwater as a water source and changes in land use, such as deforestation, reforestation and urbanization. Such operations may cause increases or decreases in flow rates (Institute of Hydrology, 1980; Smakhtin, 2001) and may well invalidate the assumptions commonly associated with the analytical tools described below and in previous sections of this chapter.

5.8.2 At-site low-flow frequency analysis

Information on low-flow frequency is obtained from an analysis relating the probability of exceeding an event to its magnitude. Such relationships can be established for low flows of various durations, such as 1, 3, 7 or 14 days or other periods or durations of interest. Commonly, non-parametric frequency analysis or probability distributions are used to describe the frequency relationship of observed seasonal or annual low flows. As in the case of flood flows, the parent distribution of low flows is unknown.

Various studies have been conducted to ascertain which distributions and which parameter estimation methods may best represent the distribution of low flows (see for example Nathan and McMahon, 1990; Lawal and Watt, 1996; and Durrans and Tomic, 2001). Results of the studies tend to differ, as the same distributions, fitting methods or data are not always used.

Matalas (1963) analysed data for 34 sites in the United States using the Pearson type III (P3), the Pearson type V (P5), the Gumbel type III (G3), which is also known as the three-parameter Weibull (W3), and the three-parameter log-normal (LN3) distributions. He concluded that the G3 and P3 distributions performed equally well and tended to outperform the other two distributions. According to Matalas (1963), the theoretical probability distribution should have a lower boundary greater than or equal to zero, and he used this as one criterion in assessing the acceptability of a distribution. Condie and Nix (1975) performed a similar analysis of data from 38 Canadian rivers using the same probability distributions as Matalas (1963). To ascertain the suitability of the distribution, they considered solutions in which the lower boundary parameter was greater than zero and smaller than the smallest observed flow. They recommended the use of the G3 distribution, with parameters estimated by maximum likelihood, followed by the method of smallest observed drought. Condie and Cheng (1982), furthering the work of Condie and Nix (1975), continued to recommend the use of the G3 distribution for low-flow frequency analysis. In the latter study, they considered a negative lower boundary to be acceptable. In such cases, they took the area of the density function from the negative lower boundary to zero as representing the probability of the occurrence of zero flows. They also verified that the lower boundary parameter was not larger than the smallest member of the sample, as certain fitting methods can provide such unrealistic results.

Tasker (1987) showed that for 20 stations in Virginia, United States, using bootstrapping that the log-Pearson type III (LP3) and G3 distributions had lower mean square errors in estimating the 7-day 10-year (Q7,10) and 7-day 20-year (Q7,20) low flows than did the Box–Cox transformations or the log-Boughton methods. Vogel and Kroll (1989) analysed the two-parameter log-normal (LN2) and two-parameter Weibull (W2) distributions fitted to data from 23 sites in Massachusetts, United States. They concluded that the W2 distribution fitted poorly, while there was no evidence to reject the hypothesis that the data were from a LN2 distribution. In addition, they analysed a variety of three-parameter distributions, namely the LN3, the LP3 and the G3. They found that the LP3 slightly outperformed the other three- and two-parameter distributions. These studies indicate that the preferred frequency distribution varies by region and there is no one frequency distribution that clearly outperforms all others.

Zaidman and others (2003) performed an analysis of 25 natural streams within the United Kingdom having more than 30 years of record. They derived data times series for durations of 1, 7, 30, 60, 90 and 365 days for each of the basins. In turn, four three-parameter distributions, namely the generalized extreme value distribution, generalized logistic distribution (GL), P3, and generalized Pareto distribution were used to fit the data for each of the series and for each duration. Goodness-of-fit tests and judgment were used to discern the results. The findings were as follows:

(a) The candidate distributions fit the observed data points very well, with little quantitative evidence to differentiate between them;
(b) Certain distributions performed better than others with the distribution type varying with duration and basin characteristics;
(c) The base flow index (Institute of Hydrology, 1980) was very useful to quantify basin geology;
(d) With regard to less permeable basins, the P3 provided the best results for shorter durations, with the generalized extreme value surpassing the P3 for longer durations;
(e) For more permeable basins, the GL provided the best results.

It has been commonly observed (Nathan and McMahon, 1990; Durrans and Tomic, 2001) that the largest flows within a series of minima are often described more effectively by a much steeper cumulative distribution curve than would be used to describe the subsequent lower flows. In response to this phenomenon, approaches have been developed to fit only the lower portion or tail of the distribution, rather than fitting the distribution to the entire sample. Nathan and McMahon (1990) noted that a transition seems to occur where the "higher frequency flows are no longer considered" as low flows but represent more "normal conditions". Approaches such as conditional probability adjustment (Condie and Cheng, 1982; Nathan and McMahon, 1990), the application of censoring theory (Kroll and Stedinger, 1996), mixture or compound parametric models (Waylen and Woo, 1987) and non-parametric frequency approaches (Adamowski, 1996; Guo and others, 1996) have been advocated to compensate for sample heterogeneity. Such approaches can also be used to perform an analysis when zero flow values are present in the sample.

Durrans and Tomic (2001) explored the performance of a number of methods that place an emphasis on fitting only the tails of the distributions. They concluded that the various methods performed "about as well as, if not better than, an estimation strategy involving fitting" the entire dataset to the LN distribution using L–moments. In contrast to this approach, for areas where annual or seasonal low-flow series may be generated by more than one mechanism and if these mechanisms can be identified, a mixture or compound parametric model could provide a more reasonable description of the data. Alternatively, non-parametric frequency estimation, as proposed by Adamowski (1996) and Guo and others (1996), could be employed. Furthermore, it has been shown that non-parametric estimation procedures provide estimates of low-flow quantiles as accurate as or more accurate than those produced by more traditional parametric approaches, namely the LP3, W2 and W3 distributions, based on simulation experiments with homogenous samples.

Low-flow statistics are generally computed for periods or durations of prescribed lengths, such as 1, 3, 7, 14, 30, 60, 90, 120, 183 and 365 days. The low-flow data for various durations are computed using a moving average for the desired period. The moving average is the lowest arithmetically averaged flow of d consecutive days within a given year. As a rule, these values are computed over a hydrological or climatic year rather than a calendar year. The hydrological year is defined to start in a season when the flow is most likely to be high so that yearly low-flow periods are not likely to be partitioned into different years. Statistics such as the mean annual d-day minimum can be computed, as can the d-day, T-year low-flow statistic, commonly denoted as Qd,T. In general, the specific d-day duration is selected according to agricultural, biological or engineering applications, which are usually related to the impact of the risk associated with the duration of low water availability on the system under study. Computational methods for estimating the parameters of the distribution of the d-day series are similar to the methods described for flood frequency analysis, with some minor variations, such as the parameter estimation method of smallest observed drought for the G3 distribution.

Two HOMS components are of particular interest for estimating low-flow frequency statistics of d-day durations. They are I80.2.03, the low-flow frequency analysis package, which allows testing of the hypotheses for randomness, homogeneity, trend and independence, and I80.2.04, Program LOWSTATS, the low-flow statistical package.

Limited analyses have been performed for durations in excess of one year and the frequency of these multi-year flows have been determined using plotting positions (Carswell and Bond, 1980; Paulson and others, 1991). Frequency analyses of multi-year low flows are important in water-supply storage analysis where carry-over storage from year to year is required to meet water-supply demands. HOMS component I80.2.05 Program DROUGHT, estimation of the probability of occurrence of n-month droughts, can be used to facilitate the analysis of multi-year events.

Examples of low-flow frequency curves for various durations are shown in Figure II.5.9. The low-flow data are typically plotted with a logarithmic or arithmetic scale for the ordinate and a normal probability scale or Gumbel scale as the abscissa. Although few data samples will plot as a perfect straight line, these types of paperare used to visually assess the overall fit of the model to the data. Special graph paper has been constructed to allow the normal and Gumbel distribution to be drawn as a straight line. Methods have also been developed to change the scale of the abscissa for various three-parameter distributions such that the cumulative distribution function will plot as a straight line (Vogel and Kroll, 1989). This change of scale would be valid for only one curve within the family of curves for the particular family of distributions, as the skewness would most likely change with duration. The technique of adjusting the abscissa to reflect sample skewness is not commonly employed in the graphical representation of low-flow results.

5.8.3 Low-flow frequency estimation at partial-record sites using base-flow measurements

The methods described thus far are valid for gauged sites having sufficient data upon which to base a

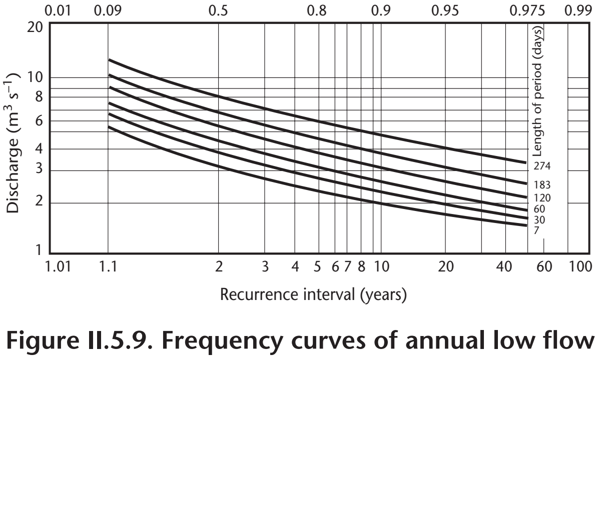


Figure II.5.9. Frequency curves of annual low flow

frequency analysis: usually 10 years or more. However, discharge measurements made at ungauged sites during times of low or base flow can be used in conjunction with concurrent daily flows at nearby gauged sites to estimate low-flow frequency. Sites where only base-flow measurements are available are referred to as partial-record sites. A relation is established between the base-flow measurements at the partial-record site and concurrent daily flows at the nearby gauged site. This relation and low-flow characteristics at the gauged site are used to estimate d-day, T-year flows at the partial-record site. The gauged site should have topographic, climatic and geological characteristics similar to the partial-record site. In order to achieve a linear relation, the logarithms of concurrent base-flow measurements $\tilde{y}$, at the partial-record site, and daily flows $\tilde{x}$, at the gauged site, are normally used to estimate the parameters of the linear relation. Such observations should be separated by significant storm events so as to represent reasonably independent observations of the low-flow processes. At least 10 paired observations are needed to define the relation between concurrent base-flow measurements $\tilde{y}$, and the daily flows $\tilde{x}$. The analysis is based on the assumption or approximation that the relation between $\tilde{y}$ and $\tilde{x}$ can be described by:

$$\tilde{y} = a + b\tilde{x} + e \quad e \sim N(0,s_e^2) \qquad (5.47)$$

Where: a, b, and s_e^2 are the constant, coefficient and variance, respectively, of the linear regression equation. It is assumed that the residuals, e, are independent and normally distributed. The estimators of the mean, $M(y)$, and variance, $S^2(y)$, of the annual minimum d-day low flows at the partial-record site are as follows:

$$M(y) = a + b\,M(x) \qquad (5.48)$$

and

$$S^2(y) = b^2\,S^2(x) + S_e^2[1 - (S^2(x)/(L-1)S^2(\tilde{x}))] \qquad (5.49)$$

where $M(x)$ and $S^2(x)$ are the estimators of the mean and variance of the annual minimum d-day low flows at the gauged site, L is the number of base-flow measurement and $S^2(\tilde{x})$ is the variance of the concurrent daily flows at the gauged site.

The primary assumption is that the relationship between instantaneous base flows, as shown in equation 5.47, is the same as the relation between the annual minimum d-day low flows at the two sites. This is a necessary assumption if the proposed method is to be used to estimate the d-day, T-year low flows at the partial-record station. While this approximation appears reasonable for d-day means up to about seven days, it may not be satisfactory for durations significantly longer than this. Stedinger and Thomas (1985), and Thomas and Stedinger (1991) discuss the use of base-flow measurements to estimate low-flow characteristics at partial-record sites in the United States.

The d-day, T-year low flow at the partial-record site is estimated using the mean and variance given in equations 5.48 and 5.49. If a three-parameter distribution is used to estimate the d-day, T-year flow at the partial record site, then the skewness of the gauged site is assumed to be the same that at the partial-record site. As described earlier, the d-day, T-year low flow at the gauged site can be estimated using procedures described in 5.8.2. Stedinger and Thomas (1985) explain why the d-day, T-year low flow at the gauged site cannot simply be used as the independent variable in equation 5.47. A loss of variance is associated with using least-squares regression equations to estimate frequency estimates such as the d-day, T-year low flow. In particular, substituting the d-day, T-year low flow at the gauged site in equation 5.47 would tend to overestimate the d-day, T-year low flow at the partial-record site. Stedinger and Thomas (1985) developed a procedure for obtaining an unbiased estimate of the variance of the annual d-day flows at the partial-record site using the relation in equation 5.49 and the variances of the annual d-day low flows and the concurrent daily flows at the gauged site.

Stedinger and Thomas (1985) also developed a procedure for estimating the standard error of the d-day, T-year low flow at partial-record stations. They illustrate that the standard error is a function of the correlation between the base-flow measurements and daily flows, the number of base-flow measurements made at the partial-record site, the magnitude of the departure of the d-day, T-year low flow at the gauged site from the mean of the daily flows used in equation 5.43 and the record length at the gauged site. Using data for 20 pairs of gauging stations in the eastern United States, Stedinger and Thomas (1985) illustrated that standard errors of about 30 percent can be achieved for partial-record stations when the correlation coefficients exceed about 0.7 and there are 15 base-flow measurements and 25 years of record at the gauged site. Using data for 1 300 gauging station in the continental United States, Reilly and Kroll (2003) demonstrated that the base-flow correlation approach gave improved results over regional regression models for 15 of the 18 major river basins in the United States. Because the method utilizes at-site data, the base-flow correlation method generally provides more accurate estimates of d-day, T-year low flows than would the regional regression models described in the next section.

5.8.4 Regionalization of low-flow frequency statistics

The methods described thus far are valid for sites having sufficient data upon which to base a frequency analysis or for which base flow measurements are available. Such sites should be relatively free of human intervention and should be of sufficient record length as to provide an accurate representation of low-flow statistics for the basin. These statistics can be estimated for ungauged basins based on regionalization methods or through the abstraction of statistics from generated time series data obtained through statistical or deterministic modeling. The first approach is most commonly used to estimate the low-flow statistic of interest, for example the seven-day, two-year low flow, Q7,2, at ungauged sites. The statistic of interest is regressed against a number of independent or explanatory variables. These independent variables represent physical and climatic characteristics of the basin. Such approaches have been used with success for the estimation of design floods, but it has been found to be much more difficult to find accurate regression models to estimate low-flow statistics (Vogel and Kroll, 1992; Waltemeyer, 2002).

Regionalization generally entails the identification of homogeneous regions over which a particular regression equation applies. Regionalization is an attempt to group basins geographically or in a multivariate space, which may not result in geographically contiguous regions, based on physiographic, climatic or streamflow characteristics. In generaly, the ability to define homogeneous regions results in increased predictive accuracy and more meaningful physical models for the statistical estimation procedure (Nathan and McMahon, 1992; Waltemeyer, 2002).

HOMS component K10.2.04, regional analyses of streamflow characteristics, describes approaches for developing regional relationships between streamflow and basin characteristics.

Regional low-flow models are generally expressed in the following form:

$$Q_{d,T} = aX_1^{\,b}X_2^{\,c}X_3^{\,d}\ldots \quad (5.50)$$

where $Q_{d,T}$ is the d-day, T-year low-flow statistic, the X_i are basin physiographic or climatic characteristics, and a, b, c and d are parameters obtained through multiple regression analysis (Weisberg, 1980; Draper and Smith, 1981). Various low-flow statistics are estimated from an at-site frequency analysis of the data from different sites within a region. Basin and climatic characteristics are, in turn, derived from maps or from climatological data (see Institute of Hydrology (1980), Vogel and Kroll (1992) and Waltemeyer (2002). The parameters of the equation can be estimated using ordinary, weighted or generalized least squares techniques. Although the technique of generalized least squares is more difficult to apply than ordinary least squares, Vogel and Kroll (1990) observed in their modeling of 23 basins in Massachusetts that the estimated parameters and the t-ratios obtained using the two approaches were almost identical. However, the more complex approach provides information on the composition of the error of prediction, allowing the attribution of error to model error, measurement error and sampling uncertainty. Vogel and Kroll (1990) noted that model error was by far the major component of the prediction error. Their analysis helps to emphasize the importance of establishing more physically meaningful statistically based model.

Regional regression equations of the form of equation 5.50 are applicable for regions where the d-day, T-year low flows are non-zero. Tasker (1991) has developed procedures for estimating low flows in regions where the d-day, T-year low flow may be zero. These techniques involve developing regional relationships with censored data and the use of logistic regression to estimate the probability of the d-day, T-year being zero.

Numerous basin and climatic characteristics have been used in regional regression equations to estimate low-flow statistics. Most models include a drainage area and a variable representing climatic conditions, such as mean annual precipitation. Many other characteristics have been considered, some of which are the mean watershed elevation, proportion of basin in forest cover, proportion of basin in lakes and swamps, average basin slope, drainage density, main channel slope and proportion of urban area. Given that low flows are normally a result of the prolonged absence of rainfall, it is commonly thought that their low-flow characteristics should be closely related to the underlying geological and soil characteristics of the basin (Institute of Hydrology, 1980; Task Committee of the Hydraulics Division, 1980). In certain cases, improved relationships have been attained by including an explanatory variable representing a geological index. Such indexes have seen increasing popularity and have led to increases in model performance. The base flow index (Institute of Hydrology, 1980) could be considered to reflect, in part, basin geology and is the ratio of flow, generally known as baseflow, to the total flow. According to Gustard and Irving (1994), a soil index can lead to improved prediction models.

Another approach has been taken to improve linkages between low-flow characteristics and recession curves or coefficients for the basin. Bingham (1982) defined a streamflow recession index, in days per log cycle of discharge depletion, at gauged streams in Alabama and Tennessee (United States) and then mapped the index according to computed indices at gauging stations and a geological map for use in estimating low-flow characteristics for ungauged streams. Vogel and Kroll (1992) formulated a conceptual model of the form of equation 5.50 to relate the unregulated flow of a basin during recession periods to the basin's characteristics. They regressed Q7,10 with three of the five variables of the conceptual model. Vogel and Kroll found dramatic increases in accuracy by inclusion of the three variables in the final regression model. The characteristics they considered were drainage area, the base flow recession constant and the average basin slope. The final equation, although accurate, cannot be used directly at an ungauged site without additional efforts being required to estimate the base flow constant. Vogel and Kroll suggest that this independent variable could be estimated from maps that would have to be developed or could be obtained through a modest and targeted streamflow gauging program. They suggest that the recession constant could be estimated simply by observing a few recession hydrographs.

Other regional low-flow studies in the United States have used soils characteristics (Carpenter and Hayes, 1996) and the slope of the flow-duration curve (Arihood and Glatfelter, 1991) as explanatory variables in estimating low-flow characteristics. Arihood and Glatfelter (1986) mapped the ratio of the 20-percent to 90-percent flow duration in Indiana for use in estimating low-flow characteristics for ungauged watersheds. Flow-duration curves are discussed in the next section of this paper.

5.8.5 **Flow-duration curves**

Flow-duration curves of daily discharge show the percentage of days that the flow of a stream is greater than or equal to given amounts over a given period of record. However, they provide no information on the temporal sequences of the flows at a site or the probability of exceedance or nonexceedance in any given year. Even with this temporal limitation, flow-duration curves have a long history of use in water resources planning and management for a variety of purposes. Some of the most common uses of flow-duration curves are in computing hydroelectric power potential for prime power and secondary power, water-supply and irrigation planning, waste-load allocations and other water-quality management problems. Other uses include the determination of wastewater-treatment-plant capacity, river and reservoir sedimentation studies, instream flow requirements for habitat management and the determination of optimal allocation of water withdrawals from reservoirs. They have also been found to be very simple and useful for graphically illustrating flow characteristics from flood to low flows for a basin. The shape of the curve can vary from basin to basin, reflecting differences in physiography and climatology. They are also useful for illustrating impacts of intervention on water availability and can be used for a host of other purposes.

A flow-duration curve is usually constructed empirically by computing a series of ratios of the number of days in a streamflow record that have discharges greater than or equal to preselected values divided by the total number of days in the record. The ratios, which are estimates of the probabilities, are plotted against their respective discharge values to construct the curve. A duration curve of streamflow will generally plot as roughly a straight line on logarithmic probability paper, such as the one shown in Figure II.5.10. This type of paper gives equal plotting accuracy at all discharges so that differences in low-flow characteristics can be discerned more precisely. Flow-duration curves are sometimes based on weekly or me. Such curves are usually less useful than a daily duration curve.

If the streamflow data are stationary, the derived flow-duration curve should provide the long-term exceedance probabilities for the entire range of flows, which is a useful planning tool. The tails of the flow-duration curve have been found to be sensitive to the number of years used to estimate the curve, which is a form of sampling error. Additional details on construction of flow-duration curves are available in other sources (see, for example, Searcy (1959), Institute of Hydrology (1980), and Vogel and Fennessey (1994).

Flow-duration curves can also be computed for each year, with the average or median of the annual-based flow-duration curves representing the typical curve (Vogel and Fennessey, 1994). These allow the development of confidence intervals and return periods to be associated with the flow-duration curve, and the resultant median annual flow-duration curve is less sensitive to extreme periods of observations that may arise over the history of a site.

The overall shape and, in particular, the shape of the lower portion of flow-duration curve is an indicator of the physiographic, geological and climatic conditions of the basin. Of most interest in low-flow studies is the shape of the lower portion of the flow-duration curve. A low-sloping lower portion implies that the basin is permeable and that the response of the basin to rainfall is not flashy. In contrast, a higher-sloping lower curve implies that the basin is less permeable and probably provides a flashy response for a given input of rainfall. A basin with a higher permeability would also tend to have a higher base flow index than the basin with lower permeability (Zaidman and others, 2003).

Regional relationships can be developed to provide estimates of flow duration for ungauged sites within a homogeneous region (Institute of Hydrology, 1980; Fennessey and Vogel, 1990; Ries, 1994). Multiple regression models similar to those outlined for the estimation of low-flow statistics, such as the Q7,10, can also be developed for this purpose. The dependent variable would be, for example, the

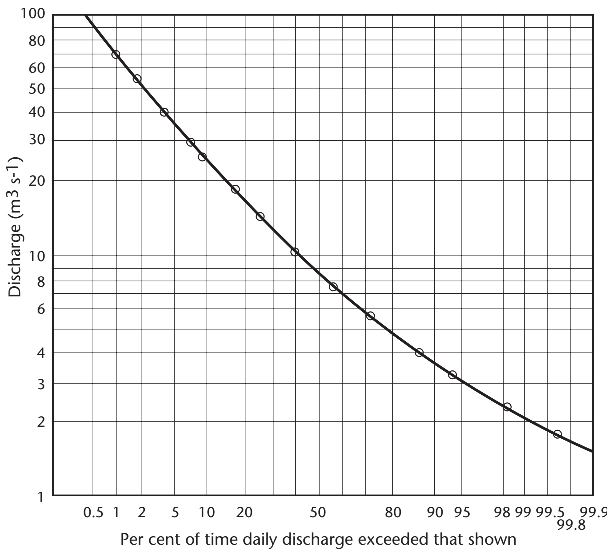


Figure II.5.10. Flow-duration curve of daily discharge

value of the flow exceeded 95 per cent of the time, denoted as Q95 (Institute of Hydrology, 1980). The independent variables of such relationships are also similar to those for other low-flow statistics and would reflect basin characteristics and climatic conditions, such as the drainage area and long-term mean annual precipitation in the basin. HOMS component K10.2.05, regionalization of flow-duration curves, or REGFLOW, can be used to estimate flow-duration curves. It can also be used to relate these to geomorphological characteristics so that flow-duration curves may be estimated for ungauged basins.

5.9 FREQUENCY ANALYSIS OF FLOOD FLOWS [HOMS H83, I81, K10, K15]

In a number of cases, for example, in storage-reservoir design, it is necessary to establish the frequency of flood volumes as well as peak flows. A multivariate statistical analysis of flood hydrographs may be used in this case. A flood hydrograph may be defined by means of the following random variables:

Qmax is the maximum discharge during the flood period; *V* is the volume (in m^3) of the flood wave; and *T* is the duration of the flood period.

By using another system of variables, a flood hydrograph may be defined by means of the sequences of discharges $Q_1, Q_2, Q_3, ..., Q_n$ corresponding to successive equal intervals of time during the flood period. Statistical analysis of the random variables (Q, V, T) or $(Q_1, ..., Q_n)$ may be performed by means of a multivariate probability distribution. Some definitions and computational techniques connected with such probabilistic models may be found in Cavadias (1990). In the case of flood characteristics, a power transformation or other methods may be used to normalize the data. Alternatively, the frequency or probability of occurrence or non-occurrence of a flood volume for an n-day period can be directly estimated by performing a frequency analysis of the site flow data or employing regionalization methods.

The purpose of computing flood and rainfall frequencies is to relate the magnitude of a flood or rainfall depth to its frequency or probability of future occurrence. The key assumptions used to allow interpretation of the frequencies as probabilities are temporal independence of the elements of the analysed sample and stationarity of the record. For flood studies, the use of partial duration series is more questionable than for rainfall, as the different peak floods during the year may be less independent than the corresponding precipitation. However, if care is taken in the selection of the values exceeding a given threshold, a partial duration series analysis should be suitable. The application of frequency analysis to a series of the annual flood maxima – maximum annual series – is more common.

The maximum annual series may be comprised of either daily maxima or instantaneous flood peaks. It is important to distinguish which of the two is required for the analysis. The relation of the two series at a site is dependent on the physical characteristics of the watershed as well as the climatologic factors causing the maxima of both events. For very small drainage areas, it is common that the two maxima do not occur on the same date nor as a result of the same climatic processes acting on the watershed.

Thus, the simplest and most straightforward approach to estimate the frequency of large floods is to use the record available at a site to fit one of the frequency distributions described in 5.1, employing an estimation procedure (see 5.5). Unfortunately, records are not always available at the sites of interest and records may be too short to provide reliable estimates of the rare floods of concern. Thus, most of the discussion in this section addresses the use of information at more than one site to estimate flood quantiles at sites which do not have flood record.

Caution must also be observed in computing frequencies of floods: a clear distinction should be made between stages and discharges. Natural changes in the stage–discharge relationship with time or direct intervention in the channel may render many stage data non-homogeneous and unsuitable for frequency analysis. For most studies, it is preferable to work with discharges, and, if necessary, to then convert the results to stage frequency using an appropriate stage–discharge relationship. In certain cases, such as high stages caused by ice jams, it may be more suitable to work solely with stages for defining flood plains because the flow rate is not an issue.

5.9.1 Regionalization of flood flows

For a site that does not have a large number of observations in its maximum annual series, regional flood frequency analysis is recommended for the estimation of the flood quantiles. Even with 50 years of data it can be very difficult to regionalize the shape parameter of a distribution. As the record gets shorter, regionalizing the coefficient of variance should be considered. However, the point at which it becomes appropriate to regionalize depends on the homogeneity of the regions that can be constructed and the relative accuracy of at-site estimators, which depends upon the at-site coefficient of variation and the skewness of the flood distribution in the region. Two popular regionalization procedures are the index flood method and the regression-based procedures; Fill and Stedinger (1998) explore the combination of the two. Regional procedures rely on data available from other stations in the same hydrologic region to obtain estimates of flood characteristics at the site of interest. Cunnane (1988) indicated that a regional approach can produce more accurate flood estimates, even when a large number of observations are available at that site. In general, there are two steps in a regional flood frequency procedure:

(a) The delineation of hydrologically homogeneous regions consisting of identification of stations with similar behaviour;
(b) Regional estimation, which involves information transfer from gauged sites to the site of interest within the same region.

Homogeneous regions in can be defined in three different ways, as illustrated by Figure II.5.11:

(a) As fixed geographically contiguous regions;
(b) As fixed geographically non-contiguous regions;
(c) As neighbourhoods, where each target station is associated with its own region.

Regional flood estimation procedures can be defined by considering various combination techniques for the determination of homogeneous regions and a number of regional estimation methods (Stedinger and Tasker, 1986; Burn, 1990; Fill and Stedinger, 1998; Pandey and Nguyen, 1999). GREHYS (1996*a*, 1996*b*) presented the results of an inter-comparison of various regional flood estimation procedures obtained by coupling four homogeneous region delineation methods and seven regional estimation methods. GREHYS (1996b) concluded that the neighborhood approach for the delineation of groups of hydrologically homogeneous basins is superior to the fixed-region approach. Hydrological neighborhoods can be determined by using the region-of-influence method (Burn, 1990) or canonical correlation analysis (Cavadias, 1990; Ouarda and others, 1998). Regional flood estimation can be carried out using the index flood method or multiple regressions.

5.9.2 Homogeneous region delineation

5.9.2.1 Region-of-influence method

The region-of-influence method (Burn, 1990), considers each site as the centre of its own region. The identification of a region of influence for a target site is based on a Euclidian distance measure in a multidimensional attribute space. The set of attributes can be related to extreme flow

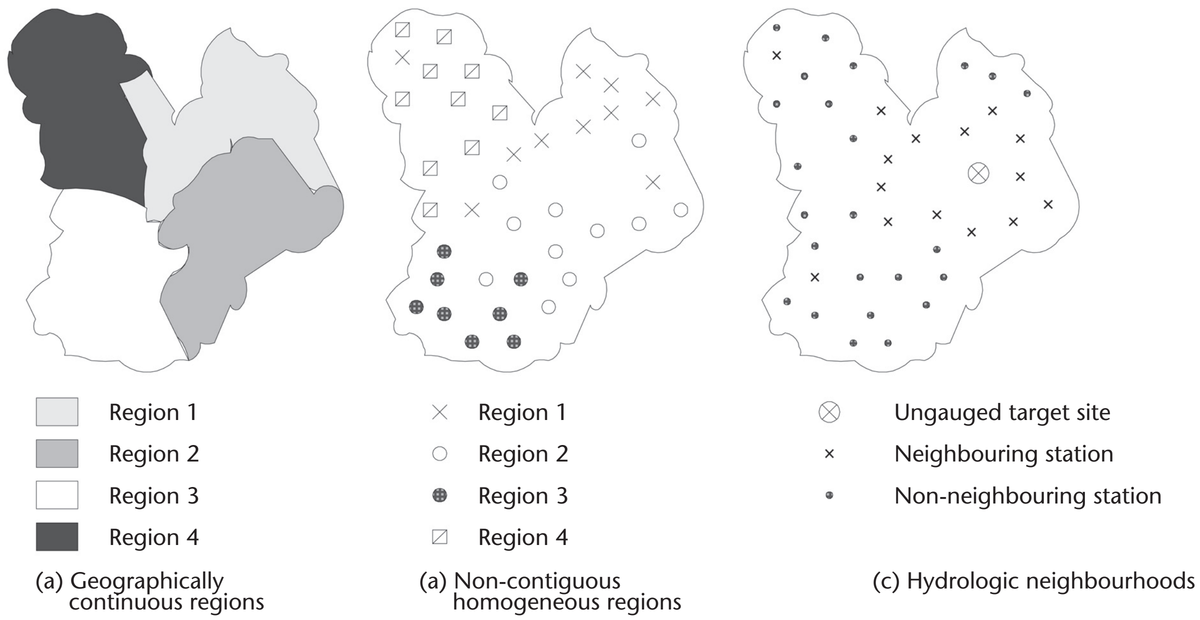


Figure II.5.11. Approaches for the delineation of homogeneous regions (Ouarda and others, 2001)

characteristics of catchments. A weight function is defined to reflect the relative importance of each site for regional estimation at the target site. In the original approach, flow attributes are used to define the region of influence, implying that the site of interest must be gauged. For ungauged sites, climatological and physiographical information may be used as a surrogate for hydrological information. Hence, several versions of the region of influence approach can be considered here, depending on whether the target site is gauged or ungauged, and depending on the space of the attributes. Hydrological attributes that can be considered are the coefficient of variation of maximum floods and the ratio of the mean maximum flow to the drainage area. Other attributes include the longitude, the latitude and meteorological attributes associated with flood events such as the mean total annual precipitation, or the mean snow depth on the ground five days before the spring flood.

The weighted Euclidian distance in the attribute space, D_{ij}, between two sites i and j is given by the following equation:

$$D_{ij} = \sqrt{\sum_{m=1}^{M} \omega_m (C_m^i - C_m^j)^2} \qquad (5.51)$$

where M is the number of attributes considered, and $C_m{}^i$ and $C_m{}^j$ are the standardized values of the attribute m for sites i and j. The attributes are standardized by division by their standard deviation over the entire set of stations. The next step is to select a threshold value, ω on D_{ij}, to define the limit of inclusion of stations in the region of influence of a target site.

5.9.2.2 Canonical correlation analysis

Canonical correlation analysis is a multivariate statistical technique that allows a reduction in the dimensionality of linear dependence problems between two groups of variables. This method can be used to identify sites with flood regimes similar to the target site (Cavadias, 1990; Ouarda and others, 1997).

Ouarda and others (1997) demonstrated that the multiple regression method and the index flood method give equivalent results when combined with the canonical correlation analysis. Ouarda and others (1999) presented an automated and transposable regional procedure based on canonical correlation analysis and multiple regressions. The general methodology presented in Ouarda and others (2000) allows the joint regional estimation of flood peaks and flood volumes. A more detailed description of the canonical correlation analysis methodology for regional frequency estimation is available in Ouarda and others (2001). A general description of the method can be found in Muirhead (1982).

5.9.3 Regional flood estimation methods

The second step of regional analysis consists in inferring flood information, such as quantiles, at the target site using data from the stations identified in the first step of the analysis. Regional estimation can be accomplished using the index-flood or regression methods.

5.9.3.1 The index-flood procedure

The index-flood procedure consists of two major steps. The first is the development of the dimensionless frequency curve for a homogeneous region. The curve is derived from individual frequency analyses of all sites. The curve for each site is made dimensionless by dividing the curve by an index, such as the flood corresponding to the two-year or 2.33-year return period or the mean. The median dimensionless values are selected for the sites for various return periods. They are in turn plotted on probability paper. The second step consists of the development of a relationship between the index and the physical and climatological characteristics of the watershed using regression-based procedures. The combination of the index with the dimensionless curve provides a frequency curve for any watershed within the region.

Much work has been done to extend these initial concepts and assess the accuracy of index procedures to determine various flood quantiles, for example in Gabriele and Arnell (1991). Advances have been facilitated by the development of probability-weighted-moment (Greanwood and others, 1979) and L–moment (Hosking, 1990) statistics. The need for analytical homogeneity tests can be circumvented by the use of Monte Carlo experiments. Homogeneity should and can be extended from the slope of the curve, which is the coefficient of variation of the sample in Dalrymple’s approach, to include the skewness and kurtosis of the proposed region. This leads to a more flexible index procedure that allows higher moments of the region’s data to indicate the potential underlying distribution. Heterogeneity of the lower moments can be assessed and potentially linked to characteristics of the watershed. Hosking and Wallis (1988) show that “even when both heterogeneity and intersite dependence are present and the form of the [regional] flood-frequency distribution is mis-specified, regional flood frequency analysis is preferable to at-site analysis". The index-flood method has been found to be one of the most efficient regionalization techniques.

5.9.3.2 Regression-based procedures

Regression techniques can be used to estimate the magnitude of a flood event that will occur on average once in *Tr* years, denoted Q_{TR}, by using physical and climatological watershed characteristics. The magnitudes of flood events for various return periods for each gauging station are estimated by using a preselected distribution from an at-site frequency analysis. In turn, characteristics for each watershed are derived from topographic maps or from generalized climatological data. The parameters of the equations that relate Q_{TR} to the characteristics can be obtained by using ordinary least squares, weighted least squares or generalized least squares techniques. The latter two approaches have been used to overcome the deficiencies in the assumptions of ordinary least squares. Ordinary least squares regression procedures do not account for variable errors in flood characteristics caused by unequal record lengths at gauging stations. Tasker (1980) proposed the use of weighted least squares regression with the variance of the errors of the observed flood characteristics estimated as an inverse function of the record length. Generalized least squares have been proposed because they can account for both the unequal reliability and the correlation of flood characteristics that exist between sites. Using Monte Carlo simulation, Stedinger and Tasker (1985 and 1986) demonstrated that the generalized least squares procedure provides more accurate estimates of regression coefficients, better estimates of the accuracy of the regression coefficients and better estimates of the model error.

The regional flood–frequency relationship developed by Benson (1962) for the north-eastern United States is as follows:

$$Q_{TR} = aA^b Z^c S^d P^e D^f M^g \tag{5.52}$$

where Q_{TR} is the T-year annual peak discharge, A is the drainage area, Z is the main channel slope, S is the percent of surface storage area plus 0.5 per cent, P is the T-year rainfall intensity for a particular duration, D is the average January degrees below freezing, M is an orographic factor, and a, b, c, d, e, f and g are regression coefficients. Many independent variables were tested to derive equation 5.52 and many definitions were tested for each variable. The goal is to obtain independent variables that are physically related to the dependent variable. Independent variables that are related to a low return-period flood may not be a driving force behind a higher return-period flood. A logarithmic transformation of equation 5.47 may be taken to create a linear additive model for the regression procedures. Other types of transformations could be applied to the dependent and independent variables, but the logarithmic transformation remains the most popular. Both the signs and the magnitude of the coefficients of the model should make hydrological sense. For example, the exponent d of the surface-storage term should be negative because of the effect of such storage (lakes, reservoirs and so forth) in flattening out flood peaks. Other exponents should be positive with their magnitudes varying with the return period. Care should be taken to ensure that not too many independent variables are included in the model. The variables included in the regression model should be statistically significant at some preselected and generally accepted level of significance (Draper and Smith, 1981).

The resulting regression equation should be evaluated to determine if it is regionally homogeneous. Residual errors of the regression should be plotted on topographic maps to check visually if geographic biases are evident. If a bias in the estimation of the T-year annual peak discharge is geographically evident, then the Wilcoxon signed-rank test can be applied to test its significance. The test provides an objective method for checking the hypothesis that the median of the residuals in a sub-region is equal to the median residual of the parent region for which the regression equation was computed. Different homogeneous regions may be found for different return periods. The homogeneous region for the relationship linking the index flood to the characteristics of the watershed need not coincide with the homogeneous region for the characteristics of the distribution of the index method, such as the slope of the dimensionless curve.

In practice, the power form function is the most commonly used model to describe the relationship between the at-site estimates of flood quantiles Q_T and the hydrometeorological and basin characteristics for the region identified in the first step of the procedure. A common procedure for the estimation of the parameters consists in lineralizing the power relationship by a logarithmic transformation, and then estimating the parameters of the linearizing model by an ordinary least squares technique. The usual procedure is therefore straightforward, because one can make use of multiple linear regression techniques to identify the parameters of a nonlinear model.

An advantage of the multiple regression regional estimation models is the flexibility in choosing the type of distribution to represent the exceedances at each site. The regression-based regional estimation method can also be applied using peaks-over-threshold data, in which case the generalized Pareto, exponential, and Weibull distributions can be used. Both the generalized Pareto distribution and the Weibull distribution contain the less flexible exponential distribution as a special case. In the peaks-over-threshold approach, all flood peaks above a prefixed threshold are considered. The lack of detailed guidelines for choosing the most appropriate threshold constitutes a serious drawback of the method and is probably one reason why it is less used in practice than its annual flood series counterpart. For a review of various methods for threshold selection, see Lang and others (1999).

A regression-based method can also be performed using non-parametric frequency analyis, which does not require a priori distribution selection. Adamowski (1989) and Guo (1991) found that non-parametric methods are particularly suitable for multimodal annual flood data following mixed distributions. Non-parametric density estimation has been used successfully in a regional framework (GREHYS, 1996*b*), including non-parametric regression (Gingras and others, 1995). As well, the L–moments technique can be used at all stages of regional analysis including homogeneous region delineation and testing, identification and testing of regional distributions and quantile estimation (Hosking and Wallis, 1997).

5.9.4 At-site and regional flow–duration–frequency approach

Most of the regional flood frequency analysis literature describes a flood event only by its instantaneous peak or its maximum daily flow. When designing a hydraulic structure or mapping a flood plain, information about flood peaks is essential, but more information may be desired. Indeed, flood severity is not only defined by the flood's peak value, but also by its volume and duration. The analysis of flow duration frequency, or QDF (Sherwood, 1994; Javelle, 2001), also known as flood duration frequency or discharge deviation frequency, has been proposed as an approach for a more thorough description of a flood event. Flow–duration–frequency analysis is similar to the intensity–duration–frequency analysis commonly utilized for rainfall (see 5.7 above). In this case, averaged discharges are computed over different fixed durations D. For each duration, a frequency distribution of maximum discharges is then analysed. Finally, a continuous formulation is fitted as a function of the return period (T) and the duration (D) over which discharges have been averaged. Javelle and others (2002) proposed a converging flow–duration–frequency model based on the assumption of convergence between the different discharge distributions for small return periods. This formulation has been successfully tested for basins located in France, Martinique and Canada.

Javelle and others (2002) have also presented a regional flow–duration–frequency approach, combining the local flow–duration–frequency formulation presented by Javelle (2001) and the index flood method outlined in 5.9.3.1, which is commonly used in regional flood frequency analysis. This regional model was developed by Javelle and others (2003) for 169 catchments in the Canadian provinces of Quebec and Ontario, and it was used to define different types of flood behaviour and identify the corresponding geographic regions. Javelle and others (2003) showed that the parameters of the regional flow–duration–frequency model provide information about the flood dynamics. Unlike the intensity–duration–frequency analysis for rainfall, flow–duration–frequency analysis remains under-utilized despite its strong potential.

5.9.5 Combination of single-site and regional data

The objective of procedures that aim to combine single-site and regional information is to improve upon at-site estimates that are based on a limited series of site data by using available information from other sites. The need for such procedures is particularly great in the estimation of extreme hydrological phenomena where a combination of limited site data and inference in the tails of probability distributions conspire to destabilize such estimators. A simple Bayesian approach presented by Fortin and others (1998) combines local and regional quantile estimates knowing the variance of estimation for each estimate. The United States has guidelines for combining at-site quantile estimates obtained by regional regression using the standard error of each (Interagency Advisory Committee on Water Data, 1982). The approach presented by Kuczera (1982) and evaluated by Lettenmaier and Potter (1985) is based on an empirical Bayes model that combines an at-site and regional variance and was shown to lead to substantial improvements in performance over procedures that only used at-site information.

Clearly, regional hydrological information should be of value in improving flood estimates, particularly with regard to the shape and characteristics of the tail of the distribution, as these are hard to resolve with limited at-site datasets. For this reason, procedures adopted in many countries employ some combination of at-site skew, as well as the at-site mean and standard deviation so as to estimate a flood frequency distribution. In certain cases, only the skew is regionalized, and the regional skew is average with at-site skew. In the United Kingdom, the general procedure is to use an index flood procedure that uses the at-site mean with a regional growth curve to define flood risk at a gauged site (Robson and Reed, 1999), so that the value of two parameters of the fitted logistic distribution are determined by regional data.

Striking the right balance between the use of regional information and at-site records to define the frequency curve is a challenge. Clearly the less data one has at a site, the less confidence one has in at-site estimation of statistics, and the more the weight that should be placed on regional information.The at-site standard deviation can also be weighted with a regional value (Kuczera, 1982; Lettenmaier and Potter, 1985) or the at-site mean and standard deviation cab be used with a regional shape estimator (Lettenmaier and others, 1987).

Region-of-influence ideas are appropriate here in defining the set of sites used for pooling. Using regional estimators of the coefficient of variation and skewness based on different spatial averaging scales in a hierarchical approach (Gabriele and Arnell, 1991) or regression to describe how a growth curve or a shape parameter varies continuously with basin characteristics (Madsen and Rosbjerg, 1997) are available options. The appropriate choice depends upon the homogeneity or heterogeneity of the region and other flood distribution characteristics, the length of the record available at different sites and the time an agency has to determine and understand those trade-offs. Stedinger and Lu (1995) illustrate some of the trade-offs among the number of regionalized parameters, the length of record and the number of sites available, regional heterogeneity and flood distribution characteristics.

5.9.6 Flood frequency analysis and climate variability

The foregoing discussion has for the most part embodied the traditional assumption that flood series are a set of independent and identically distributed random variables. If they are not entirely independent but instead have some modest correlation from year to year, it has relatively little impact on the analysis and the bias of estimated flood quantiles. The more troubling concern is either a trend in the distribution of floods due to development and other changes in the basin, or what has been called climate variability and climate change. All three of these effects can have a significant impact on flood risk in a basin.

The easiest of the three to deal with is when changes in the basin – particularly land cover, the drainage network and channel characteristics – or the construction and operation of detention structures have evolved over time. A traditional record of annual maximum floods is no longer effective in describing the risk of flooding under the new regime. The traditional approach to handling changes in channel characteristics and the operation of storage structures is to route a historical record of natural flows through a hydraulic model to generate a record of regulated flows, which can be used as a basis for frequency analysis. Alternatively, a frequency analysis can be conducted on the natural flows and a design natural flow hydrograph can be constructed that is, in turn, routed through the hydraulic model based on the assumption that owing to operation of the facility, the exceedance probability of the design hydrograph would be unchanged because smaller and larger events would have resulted in smaller and larger flood peaks downstream, respectively.

For complicated systems involving several streams or storage facilities, or for basins that have experienced significant land-cover and land-use change, it is advisable to use historical or synthetic rainfall and temperature series to drive physically based rainfall-runoff and hydraulic models. Such a study allows the analyst to appropriately describe the operation of different facilities, network and channel modifications, as well as the likely effect of land-cover and land-use changes.

Dealing with climate variability and climate change is a difficult problem (Jain and Lall, 2001). NRC (1998) makes the following observation:

> Evidence accumulates that climate has changed, is changing and will continue to do so with or without anthropogenic influences. The long-held, implicit assumption that we live in a relatively stable climate system is thus no longer tenable.

Changes in hydroclimatological variables, both rainfall and runoff, over different timescales are now well documented for sites around the world (Hirschboeck and others 2000; Pilon and Yue, 2002; Pekarova and others, 2003). Two cases are immediately clear, those corresponding to climate variability and climate change.

The first concern, climate variability, relates to such processes as the El Nino-Southern Oscillation or the North Atlantic Oscillations, which result in a sporadic variation in the risk of flooding over time on the scale of decades. In cases where the record is relatively, it would be hoped that such phenomena would have passed through several phases resulting in a reasonable picture of the long-term average risk. With short records, such variations are more problematic. It is always good practice to attempt to use longer records from the same region in order to add balance to the short record. If a composite or a cross-correlation between the short record and longer records in the region is reasonably high, record augmentation methods described in 5.5.4 can be used to develop a more balanced, long-term description of flood risk. However, with smaller catchments where year-to-year events are highly variable, it may not be effective to use simple record augmentation to correct distinct differences in flood risk between different periods because the cross-correlation between concurrent annual peaks will be too small.

For operational concerns, an option would be to forecast variations in the El Nino-Southern Oscillation, or other indices, and unexplained hydrological variations, so as to forecast more accurately the flood risk in the current and subsequent years and advise water operations accordingly (Piechota and Dracup, 1999). However, for project planning purposes such short-term variations are likely to be too short lived to affect the economic design of projects.

The second climate concern would be climate change in one direction or another that is not quickly reversed within a decade or two. Such climate change is on the scale of decades and is a very serious concern. Even mild upward trends can result in substantial increases in the frequency of flooding above a specified threshold, as shown by Porparto and Ridolfi (1998) and Olsen and others (1999). It is clear that anthropogenic impacts are now inevitable. The question is how soon and how severe. Guidance is much more difficult to provide for this case because there is no clear consensus on how fast the Earth is likely to warm from the release of greenhouse gases into the Earth's atmosphere and what the impact of those changes will be on meteorological processes at a regional or watershed scale. Generalized circulation models of the Earth's atmosphere have given some vision of how local climates may change, but the inability of such models to capture current meteorological processes at a regional or watershed scale yields limited confidence that they will be able to predict accurately the rate and intensity of future change. However, the hydrological implications of different generalized circulation model scenarios are often investigated to provide a vision of what the future may hold (see Arnell and others (2001)). And, as Arnell (2003) points out, the future will be the result of both natural climate variability and climate change.

5.10 ESTIMATING DESIGN FLOODS [HOMS K10, K15, I81, K22]

5.10.1 General

The design flood is defined as the flood hydrograph or the instantaneous peak discharge adopted for the design of a hydraulic structure or river control after accounting for political, social, economic and hydrological factors. It is the maximum flood against which the project is protected; its selection involves choosing safety criteria and estimating the flood magnitude that meets the criteria. The risk of damage occurring is equivalent to the probability of occurrence of floods larger than the design flood. The decisive factor in the determination of a design flood is that feature or parameter of the flood that can be identified as the major cause of potential damage. The decision as to which is the most relevant flood parameter for a particular project lies with the planner and the designer and should be based on an engineering analysis of the given situation. Decisive parameters usually include the following:

(a) Peak discharge in the case of culverts, storm sewers, bridge openings, spillways and outlets of weirs and small dams;
(b) Peak stage in the case of levees, clearance under bridges, flood-plain zoning and the design of roads and railways in river valleys;
(c) Flood volume for the design of flood-control reservoirs and, generally, for all cases where flood attenuation by water storage can be significant, such as for the design of spillway capacities and freeboards on dams;
(d) Flood hydrograph shape in cases where superposition of several floods must be considered, such as for flood protection downstream from the mouth of large tributaries or for reservoir operation during floods.

5.10.2 Design floods

The following types of design flood are commonly used in water-resource engineering practice (Singh, 1992):

(a) Spillway design flood – a term often used in dam design to identify a flood that a spillway must be able to pass to provide the desired degree of protection for a dam;
(b) Construction flood – a flood for which reasonable precautions will be taken to avoid flooding of construction sites and thereby to prevent damage to a project during its construction;
(c) Probable maximum flood – the largest flood that may be expected at a site, taking into account all pertinent factors of location, meteorology, hydrology and terrain (see 5.7). It essentially has an infinite return period and can be selected as the design flood to prevent a major disaster;
(d) Standard project flood – a flood resulting from the most severe combination of meteorological and hydrological conditions that are considered reasonably characteristic of the geographical region involved, excluding extremely rare combinations. It has a long but unspecified return period and may be selected as a design flood for structures of great importance;
(e) Frequency-based flood – a flood determined employing frequency analysis of flood flows or rainfall data by performing one of the following:
    (i) frequency analysis of rainfall data to estimate a frequency-based design storm, which is then converted to design flood;
    (ii) frequency analysis of flood flows available at the site to directly estimate the design flood;
    (iii) regional frequency analysis to estimate the design flood.

5.10.2.1 Magnitude and methods of computation

A design flood can be estimated by transforming the design storm to design flood using, for example, the unit hydrograph concept or flood frequency analysis. The latter requires long-term streamflow data at the site of interest. If streamflow data are unavailable or a hydrograph is required, then the design flood can be estimated using either a rainfall frequency analysis coupled with a rainfall-runoff model or a rainfall-runoff method that may be either data-based, or hypothetical or empirical. The rainfall information used for design flood estimation is referred to as the design storm and can be classified as probable maximum precipitation, a standard project storm, or a frequency-based storm. For structures involving low-damage risk, such as culverts and secondary roads, the design flood may be calculated by empirical methods, given the typically low return period of such structures and their relatively low importance. For structures or projects involving major potential damage, but without a risk of loss of life, design floods should be computed, if possible, by methods allowing an evaluation of their return periods so that optimization methods can be used for the selection of the design flood magnitude. For situations involving a risk of loss of life, the aim is to provide maximum protection, and the maximum probable flood or the standard project flood is usually adopted as the design flood. It is advisable to evaluate the reasonableness of the probable maximum flood by comparing it with observed rainfalls and floods.

Only a few of the more practical and popular methods for calculating floods have been described in this chapter. There are many other methods, some of which have been developed for particular regions, such as those described by Maidment (1993) and Kundziewicz and others (1993). For example, the GRADEX method (Guillot, 1993; Ozga-Zielinski, 2002) is based on the combined use of rainfall and flow records. It assumes that the upper tail of the flood is near an exponential asymptote (gradient) of rainfall. The *Flood Estimation Handbook* proposes a procedure developed by the Centre for Ecology and Hydrology in the United Kingdom that combines statistical analysis and modelling of precipitation time series to the hydrological simulation of discharge at catchment scale (www.nerc-wallingford.ac.uk).

5.10.2.2 Design life of a project and design criteria

In the wide range of cases in which the design flood is selected by optimizing the relation between the expected flood damage and the cost of flood-protection measures, the resulting optimum level of the calculated risk depends to a certain degree on the length of the period over which the performance of the project is evaluated. This period is called the design life or planning horizon of the project and is determined in the project-planning stage on the basis of the following four time spans:

(a) Physical life, which ends when a facility can no longer physically perform its intended function;
(b) Economic life, which ends when the incremental benefits from continued use no longer exceed the incremental cost of continued operation;

(c) The period of analysis, which is the length of time over which a facility may be expected to function under conditions that can be relatively accurately foreseen at the time of the analysis; any operation in the distant future that is subject to a high degree of uncertainty is excluded from consideration;
(d) The construction horizon, which is reached when a facility is no longer expected to satisfy future demands, becoming functionally obsolete.

The optimum level of calculated risk, hence the design return period for each of these periods may be different. The final selection of the design flood cannot be made without considering political, social, environmental and other quantifiable criteria.

In many cases, flood analysis criteria are often prescribed by regulations and not subject to negotiation. Different types of projects may require different types of criteria reflecting economic efficiency and safety. Safety criteria can be specified in terms of a return period, meteorological input and/or the maximum flood on record. The return period (T), in years, that is to be used is often specified by the competent agency and may be related to specified risk (R) or probability of failure (per cent) over the service life (n) (in years) as given by $T = 1/[1 - (1–R)^{1/n}]$ (see 5.10.8).

For example, when n = 2 and the acceptable risk is $R = 0.02$ per cent, then $T = 99.5$ years. A distinction should be made between specifying the criteria that is to be met and specifying the computational method to be used to estimate the design flood. When the computational method is not specified by the regulation, it must be selected and justified by the designer. It is advisable to ensure the adequacy of the design against given conditions and intent of the project.

5.10.2.3 Design floods for large reservoirs

The selection of design floods for the spillway design of large storage reservoirs must be given special attention because a reservoir may considerably change the flood regime, both at the reservoir site and in the downstream section of the river.

The basic flood-related effect of a reservoir is flood attenuation. Its estimation requires knowledge of the original flood hydrograph shape. When the hydrograph is not known, a hypothetical shape, often triangular, is assumed and fitted to the selected flood volume and peak discharge. In evaluating the effect of flood attenuation on the reduction of spillway capacity and freeboard of a dam, it is imperative to adopt a conservative approach and to consider only those effects that can be guaranteed at all times. Thus, only the effect of the ungated spillway should be considered. All gated outlets should be assumed to be closed and the reservoir filled to the crest of the fixed spillway at the beginning of the flood.

In addition to flood attenuation, the flood regime downstream must be analysed carefully from the point of view of changes in the timing of flood peaks, the effect of changes in the shape of flood hydrographs and the effects on the river channel caused by an increased scouring tendency of the virtually sediment-free water leaving the reservoir through the spillway.

The type of dam structure must also be considered because it is of prime importance in determining the vulnerability of the dam should overtopping occur. Vulnerability is highest for earthfill dams, which are in great danger of collapsing if overtopped.

5.10.2.4 Probable maximum flood

Probable maximum flood is computed from probable maximum precipitation (see 5.7) or from the most critical combination of maximum snowmelt (see 6.3.4) and rainfall, and it provides an indication of the maximum possible flood that could reasonably be expected for a given watershed. It is not possible to quantify the term reasonable or assign a long but arbitrary return period to the probable maximum flood. The concepts of probable maximum precipitation and probable maximum flood are controversial. Nevertheless, it is necessary to assess the potential impact of such extreme events; therefore, numerical flood estimates are required for very extreme floods and are often used in design practice.

Probable maximum precipitation is analytically estimated as being the greatest depth of precipitation for a given duration that is physically plausible over a given watershed at a certain time of the year, and its estimation involves the temporal distribution of rainfall. The concepts and related methodologies are described by WMO (1986*a*). The US Army Corps of Engineers (1985) has a computer program, HMRS2, to compute probable maximum precipitation, which can then be used with HEC-1 (see 5.10.5) to determine probable maximum flood. WMO (1969) provides more details on estimation of maximum floods (see 6.3.2).

As rainfall usually accounts for a major portion of probable maximum flood runoff, special consideration must be given to the conversion of rainfall to runoff. This conversion is done by deterministic rainfall-runoff models, but their application for this purpose involves certain modifications designed to accommodate the extreme magnitude of the rainfall event that is being used as input. The most important modifications are as follows:

(a) The effect of the initial soil-moisture conditions and of the variation of the infiltration rate during the rainfall on streamflow is greatly reduced, compared to their effect in streamflow simulation under normal conditions. Hence, the refined methods employed in most models for estimating infiltration indices can be considerably simplified. A common practice is to use the minimum infiltration capacity, or the maximum runoff coefficient, for a given soil type and vegetation cover, throughout the entire storm;
(b) When a unit hydrograph is used to transform the maximum precipitation, it should be remembered that the validity of the underlying assumption of linearity is limited to conditions similar to those for which the unit hydrograph was derived. An analysis of floods in a number of basins (Singh, 1992) has shown that the peak ordinates of unit hydrographs derived from major floods (greater than 125 mm of runoff over the basin area) are often 25 to 50 per cent higher than peak ordinates derived from minor floods (25 to 50 mm of runoff). It is important to bear in mind that the adjustment of the unit hydrograph for the computation of the probable maximum flood must be guided by the necessity of making a conservative estimate: one that leads to the greater flood;
(c) In the case of drainage basins larger than 500 km^2, or even smaller basins where their different parts have widely different runoff characteristics, it is generally necessary to derive separate unit hydrographs and probable maximum floods for several sub-areas and to obtain the probable maximum flood for the whole basin by routing the component floods downstream to the project site. It must be remembered that the same positioning of the isohyetal pattern of the design storm over the catchment, which yields the maximum flood if a single unit hydrograph is used for the whole catchment, need not yield the maximum flood if the catchment is subdivided into several sub-areas. Thus, for each different catchment subdivision, an optimal positioning of the design storm, that is, the position yielding the most unfavourable combination of the relevant parameters of the probable maximum flood, must be found separately subject to the restrictions due to orography, as discussed in 5.7. The optimal position of the design storm can be obtained as a result of sensitivity analysis.

Although no specific return period can be assigned to the probable maximum flood, its parameters should be compared with the respective frequency curves fitted to historical floods to make sure that they have extremely long return periods and have been unequalled by any historical flood event.

5.10.2.5 Standard project flood

A standard project flood is usually about 50 per cent of a probable maximum flood (Singh, 1992). Its determination is governed by considerations similar to those relevant to the probable maximum flood. The standard project flood is usually determined by the transformation of the transposed largest rainstorm observed in the region surrounding the project, rather than from a meteorologically maximized rainstorm, as in the case with the probable maximum flood. Nonetheless, the standard project flood should represent a very rare event and should not be exceeded by more than a few per cent by the major floods experienced within the general region.

5.10.3 Data preparation

Basic data for determining design floods are the records collected by regional or national Hydrological and Meteorological Services. These data exist in the form of stage recordings and discharge measurements that form the basis for the computation of rating curves. As the magnitude of the design flood depends primarily on measurements of high discharges, special attention should be given to their evaluation and the extension of rating curves.

For a proper assessment of the flood regime, it is essential to obtain sufficient information on historic floods. The basic element of such information is stage. In compiling information on flood stages, use can be made of traces of materials deposited by floods, flood marks on bridges, buildings and river banks; recollection of long-time residents; photographs taken during floods; archived materials; articles in the press and memoirs. Paleoflood information can also be considered (Viessman and Lewis, 2003).

To convert flood stages determined by such investigations into discharges, hydraulic computations must be based on reconstructed river cross-sections, longitudinal profiles, the slope of water surface and channel roughness. All the known modifications of the river channel should be taken into account, such as dredging, embankments and channel straightening. Owing to the limited accuracy of the reconstructed river characteristics, the application of the Manning and Chézy formulae is generally satisfactory for hydraulic computations of this kind. Software such as HEC-RAS can facilitate the analysis.

5.10.4 Design flood computation techniques

The selection of computational techniques for the determination of design floods depends on the type, quantity, and quality of available hydrological data, as well as the type of design flood information. Owing to the complexity of the flood producing process, the estimates are only approximations, and understanding of related issues is important to produce reliable estimates. There are many methods, and the choice is often made on a subjective and intuitive basis. Some practical criteria for the choice of the method can be found in Pilgrim and Doran (1993) and details of many methods are available in Pilgrim and Cordery (1993), Bedient and Huber (2002) and Viessman and Lewis (2003).

Depending on data availability and design requirements, the methods of estimating design floods can be grouped into empirical, frequency-based and rainfall-runoff methods.

To extract maximum information from scarce or inaccurate data, it is advisable to apply several different methods, compare the results and choose the design parameters based on engineering judgment. Sensitivity analysis can be useful in making the final decision because it may show the impact of potential errors on the magnitude of the design variable.

5.10.4.1 Empirical methods

Empirical flood formulae expressed as a flood envelope curve may be used to provide a rough estimate of the upper limit of discharge for a given site. A common type of formula expresses the peak discharge Q ($m^3\ s^{-1}$) as a power function of catchment's area A (km^2) (Bedient and Huber, 2002),

$$Q = CA^n \tag{5.53}$$

where coefficient C and exponent n vary within wide limits and the values for a particular study can be selected on the basis of empirical data.

The application of empirical formulae is generally limited to the region for which they have been developed, and they should be used with great caution and only when a more accurate method cannot be applied. Another drawback of empirical formulae is the difficulty in assessing the return period of the computed peak flow.

An envelope curve enclosing maximum observed peak flows can be plotted against catchment areas for a large number of stations in a meteorologically and geomorphologically homogeneous region. Such curves provide useful information, especially in cases where few data are available at any single station. Attempts have been made to refine the technique by constructing various envelopes related to different climatological and/or geomorphologic factors. However, the return periods of the peak flows remain unspecified. Uses of such formulae provide a rough estimate providing only the order of magnitude of large flood flows.

5.10.4.2 Rainfall-runoff models

Depending on whether the design flood is to be synthesized from precipitation and/or snowmelt or from known flood hydrographs at upstream points, the models of interest fall into two broad categories:

(a) Rainfall-runoff models, as described in 6.3.2;
(b) Streamflow routing models, as described in 6.3.5.

Many rainfall-runoff relationships have been developed that could apply to any region or watershed under any set of conditions. However, these methods should be used with caution, as they are only approximate and empirical. The most widely used practical methods are the unit hydrograph method (see 6.3.2.3), the rational method (see below), the Soil Conservation Service (SCS) method (see below) and conceptual models (see 5.10.5).

5.10.4.2.1 *Rational method*

One of the oldest and simplest rainfall-runoff formulae is the rational formula, which allows for the prediction of peak flow Q_p ($m^3\ s^{-1}$) from the following equation:

$$Q_p = 0.278CiA \qquad (5.54)$$

where C is the runoff coefficient that is dimensionless and selected according to the type of land use in the watershed, i is rainfall intensity (mm/hr) of chosen frequency and for duration equal to the time of concentration, and A is the watershed area (km^2). This method is often used in small urban areas as well as for rough estimates in rural areas in the absence of data for other methods. It is highly sensitive to rainfall assumptions and the selection of C. Use of this method should be restricted to small areas; although the upper limit is not explicitly established, it varies between 40 ha and 500 ha.

Because of its predominant use in urban areas, the rational method is dealt with in more detail in 4.7.

5.10.4.2.2 *Soil Conservation Service method*

The former US Department of Agriculture Soil Conservation Service, now the National Resource Conservation Service, suggested an empirical model for rainfall abstractions based on the potential for the soil to absorb a certain amount of moisture. On the basis of field observations, the potential storage S was related to a curve number CN varying between 0 and 100, which is a characteristic of the soil type, land use and the initial degree of saturation known as the antecedent moisture condition (AMC). The value of S is defined by the empirical expression:

$$S = 25.4\left(\frac{1000}{CN} - 10\right)\text{(millimetres)} \qquad (5.55)$$

The values of CN are given in Table II.5.8 as a function of soil type (A, B, C, D), land use, hydrological condition of the watershed and antecedent moisture condition (AMC I, II, III).

According to this method, the depth of surface runoff is given by the following equation:

$$Q = \frac{(P - I_a)^2}{(P - I_a) + S} \qquad (5.56)$$

where Q is the depth of surface runoff, P is the accumulated depth of rainfall, I_a is an initial abstraction: no runoff occurs until accumulated rainfall exceeds I_a, and S is the potential storage in the soil.

All units are in mm, and for values of $P > I_a$. Using observed data, the Natural Resources Conservation Service found that I_a is related to S, and on average is assumed to be $I_a = 0.2S$; thus the equation becomes:

$$Q = \frac{(P - 0.2S)^2}{P + 0.8S} \qquad (5.57)$$

for $P > 0.2S$, and $Q = 0$ when $P \leq 0.2S$. Since initial abstraction consists of interception, depression storage and infiltration prior to the onset of direct runoff, the value of I_a can be modified to account for local conditions.

Soils are classified as A, B, C, and D according to the following criteria:

(a) Group A soils have low runoff potential and high infiltration rates, greater than 7.6 mm/hr, and consist primarily of deep well-drained sands and gravel;
(b) Group B soils have moderate infiltration rates (3.8–7.6 mm/hr) and consist primarily of moderately fine to moderately coarse textured soils, such as loess and sandy loam;
(c) Group C soils have low infiltration rates (1.27–3.8 mm/hr) and consist of clay loam, shallow sandy loam and clays;
(d) Group D soils have high runoff potencial and low infiltration rates (less than 1.27 mm/hr) and consist primarily of clays with high swelling potential, soils with a permamnt high water table or shallow soils over nearly impervious material.

CN values for urban and composite areas should be determined.

The runoff from a particular rainfall event depends on the moisture already in the soil from previous rainfall. The three antacedent moisture conditions are as follows:

(a) AMC I – Soils are dry but not to wilting point;
(b) AMC II – Average conditions;
(c) AMC III – Heavy rainfall or light rainfall with low temperature have occured within the last five days saturating the soil.

Table II.5.8 provides CN(II) values for average conditions AMC II. CN(I) and CN(III) corresponding to AMC(I) and AMC(III) can be estimated from:

$$CN(I) = 4.2CN(II)/(10 - 0.058CN(II)) \qquad (5.58)$$

and

$$CN(III) = 23CN(II)/(10 + 0.13CN(II)) \qquad (5.59)$$

Table II.5.8. Runoff curve numbers for selected agricultural, suburban and urban land use (AMCII, and I_a = 0.25) (after Bedient and Huber, 2002)

Land-use description		*Hydrological soil group*			
		A	*B*	*C*	*D*
Cultivated land[a]					
Without conservation treatment		72	81	88	91
With conservation treatment		62	71	78	81
Pasture or rangeland					
Poor condition		68	79	86	89
Good condition		39	61	74	80
Meadow					
Good condition		30	58	71	78
Wood or forest land					
Thin stand, poor cover, no mulch		45	66	77	83
Good cover[b]		25	55	70	77
Open spaces: lawns, parks, golf courses and so forth					
Good condition: grass cover = 75% or more		39	61	74	80
Fair condition: grass cover = 50–75%		49	69	79	84
Commercial and business areas (85% impervious)		89	92	94	95
Industrial districts (72% impervious)		81	88	91	93
Residential[c]					
Average lot size	Average % impervious[d]				
1/8 acre[e] or less	65	77	85	90	92
1/4 acre	38	61	75	83	87
1/3 acre	30	57	72	81	86
1/2 acre	25	54	70	80	85
1 acre	20	51	68	79	84
Paved parking lots, roofs, driveways and so forth[f]		98	98	98	98
Streets and roads					
Paved with curbs and storm sewers[f]		98	98	98	98
Gravel		76	85	89	91
Dirt		72	82	87	89

a For a more detailed description of agricultural land-use curve numbers, please refer to *National Engineering Handbook* (Natural Resources Conservation Service, 1972).
b Good cover is protected from grazing and litter and brush cover soil.
c Curve numbers are computed assuming that the runoff from the house and driveway is directed toward the street with a minimum of roof water directed to lawns where additional infiltration could occur.
d The remaining pervious areas (lawns) are considered to be in good condition for these curve numbers.
e 1 ha = 0.404687 acre
f In some warmer climates of the country a curve number of 95 may be used.

5.10.4.2.3 ***Soil Conservation Service unit hydrograph***

The earliest Soil Conservation Service method assumed that a hydrograph is a simple triangle, as shown in Figure II.5.12, with rainfall duration D (hours), time to peak TR (hours), time of fall B (hours) and the peak discharge Q_p(m^3 s^{-1}) given by the following equation (Bedient and Huber, 2002):

$$Q_p = \frac{0.208\ AQ_R}{T_R} \tag{5.60}$$

where A is the watershed area (km^2) and Q_R indicates the runoff depth for unit hydrograph calculations (mm). Figure II.5.12 shows that the time to peak (hours) is as follows:

$$T_R = D/2 + t_p \tag{5.61}$$

Where D is the rainfall duration (in hours) and t_p is the lag time (in hours) from centroid of rainfall to Q_p(m^3 s^{-1}). Lag time t_p is estimated from any one of several empirical equations used by the SCS, such as:

$$t_p = \frac{l^{0.8}\,(S+1)^{0.7}}{1900\,y^{0.5}} \tag{5.62}$$

where l is the distance to the watershed divide (in feet), y is the average watershed slope (per cent) and

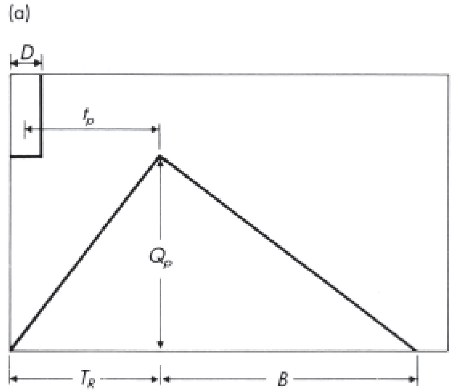


Figure II.5.12. SCS traiangular unit hydrograh

S and CN are obtained from Table II.5.7. The basin lag (t_p) is applicable to CN values between 50 and 95, and watershed areas less then 8 km^2. For urban areas, t_p should be adjusted for imperviousness. The coefficient 0.208 in equation 5.60 is an average value for many watersheds. It may be reduced by about 30 per cent for flat or swampy watersheds, or increased by about 20 per cent for steep basins. When such a change is introduced, then the unit hydrograph must also be adjusted accordingly.

Once Q_p and t_p are estimated, the unit hydrograph can be graphed and/or tabulated using the dimensionless unit hydrograph shown in Table II.5.9. Muzik and Chang (2003) developed a regional dimensionless hydrograph.

The SCS method is widely used (Byczkowski, 1996; Maidment, 1993) because of its simplicity, readily available watershed information, ease of application, and because it gives reasonable results. However, the results of studies comparing prediction with measured data have been mixed (Dingman, 2002) and so the method should be used with caution.

5.10.5 Flood hydrograph conceptual models

Recent advances in computer technology and theoretical hydrological developments have revolutionized the manner in which computations are now routinely performed. Hydrologic models allow for parameter verification in space and time, use of remotely sensed data and application of geographical information systems. Advanced computer-based technologies such as spreadsheets, databases and graphical capabilities facilitate the flexibility of data entry procedures.

Some of the more widely used models that have also been developed include:

(a) HEC-1 which was developed and is maintained by the US Army Corps of Engineers Hydrologic Engineering Center (www.hec.usace.army.mil). This model simulates the watershed as a series of hydraulic and hydrological components and calculates runoff from single storms. The user can select from a variety of sub-models that simulate precipitation, infiltration and runoff, as well as a variety of techniques to perform the flow routing. The model also includes dam safety and failure analysis, flood damage analysis and parameter optimization. More recent improvements include consideration of radar rainfall as input and the use of geographical information system and mapping tools (HEC-GeoRAS) for handling output and data manipulation;

(b) SCS-TR 20 (for agricultural watersheds) and SCS-TR 55 (for urban watersheds) were developed and are maintained by the Natural Resources Conservation Service, US Department of Agriculture. This combined model uses a curve number (CN) method to calculate the runoff hydrograph resulting from a single storm from sub areas and routed through drainage systems and reservoirs;

(c) SWMM was developed and is maintained by the US Environmental Agency (www.epa.gov/cdnnrmrl/models/swmm). This model consists of a runoff module, a transport module and a storage/treatment module. It simulates runoff quantity and quality, routes sewer flows, computes hydraulic head and simulates the effects of detentions basins and overflows. It is the most comprehensive model for handling urban runoff.

There are certainly many other good models that can perform the same tasks. Model capabilities change rapidly and therefore it is advisable to seek

Table II.5.9. Ordinates of the Natural Resources Conservation Service Dimensionless Unit Hydrograph

t/T_R	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.4	4.6	5.0
Q/Q_p	0	0.310	0.930	0.930	0.560	0.280	0.147	0.077	0.029	0.003	0.000

current information through the Websites of various model developers. Links to other popular models are www.wallingfordsoftware.com, www.dhi.dk, http://water.usgs.gov/software/lists/surface_water and www.haested.com.

All of the above models can be run on microcomputers and some are proprietary. Bedient and Huber (2002) provided a more comprehensive list of many internet sources to operational computer models, but many more have certainly become available in the intervening years.

5.10.6 Snowmelt contribution to flood

In some regions of the world, floods are caused by a combination of snowmelt and rainfall runoff or snowmelt alone. Factors affecting the contribution of snowmelt to floods include accumulated snow pack depth at time of melt, ice jamming, basin storage and the return period of the event in question. Synthesis of runoff hydrographs associated with snowmelt requires empirical equations, since snowmelt is not measured directly.

After the depth of melt has been estimated, it can be treated like rainfall and converted into streamflow by application of the unit hydrograph or routing technique. Such a procedure does not provide the probability of occurrence of a flood. Comparison of several snowmelt runoff models is described by WMO (1986*b*). There are several operational models that have a snowmelt routine, including HEC-1 (USACE, 1985).

5.10.7 Calculating discharges from urban drainage systems

Urban hydrology is concerned mainly with the prediction of runoff peaks, volumes and complete hydrographs anywhere in the system. The solution to the above problems requires various analytical methods. Peak volumes can be obtained from simplified techniques such as the rational method (see 5.10.4.2.1), while hydrographs usually require more comprehensive analysis including the Natural Resources Conservation Service method (see 5.10.4.2.2), or computer models (see 5.10.5). Urban drainage is discussed in more detail in 4.7.

5.10.8 Risk

The probability that the design flood will be exceeded at least once is known as the risk of failure, and the probability that the design flood will not be exceeded is referred to as the reliability. One of the main concerns in design-flood synthesis is an evaluation of the risks associated with the occurrence of floods higher than the design flood. Knowledge of these risks is important because of their social, environmental and economic implications, for example in the determination of flood-insurance rates, flood-zoning policies or water quality conservation. As floods are stochastic phenomena, their magnitude and the time of their future occurrence cannot be predicted. The only possibility is to assess them on a probabilistic basis, that is, to assign a probability to the possibility that a flood of a given magnitude will be exceeded within a specific period of time. A variable that has a probability of exceedance p has a return period $T = 1/p$.

Guidance for general frequency analysis is provided in 5.3, and in 5.9 for flood frequency analysis. A comprehensive risk assessment procedure for natural hazards is provided in *Comprehensive Risk Assessment for Natural Hazards* (WMO/TD-No. 955).

The probability of exceedance of a given magnitude of event, as derived from a probability distribution model, pertains to each future event. Thus, if an annual flood series is considered, the exceedance probability p defines the risk that the given magnitude will be exceeded in any one year. However, it is often necessary to calculate a probability p_n that a given event, for example the exceedance of a particular flood peak, will occur at least once in n years, for example, during the design life of a project. If the assumption of independence of floods in individual years is satisfied, this probability is:

$$p_n = 1 - (1 - p)^n = 1 - \left(1 - \frac{1}{T}\right)^n \qquad (5.63)$$

where T is the return period. This measure of risk provides a more probabilistic indication of the potential failure of the design than that encapsulated in the concept of return period. Note that the risk of an event occurring at least once during its return period follows from equation 5.63 for n equal to T. When T is large, this risk approaches the asymptotic value:

$$1 - e^{-1} = 0.63 \qquad (5.64)$$

From equation 5.63, it is possible to express T as a function of n and p_n, that is, to calculate a return period such that the risk of occurrence of the event during a period of n years will have a specified value p_n. This return period is called the design return period T_d and is as follows:

$$T_d = 1/[1 - (1 - p_n)^{1/n}] \qquad (5.65)$$

Table II.5.10. Required design return period T_d of an event whose risk of occurrence in n years is equal to p_n

	n year			
p_n	*2*	*10*	*50*	*100*
0.01	199.0	995.0	4975.0	9950.0
0.10	19.5	95.4	475.0	950.0
0.50	3.4	14.9	72.6	145.0
0.75	2.0	7.7	36.6	72.6

Some values of the variables T_d, n, and p_n are shown in Table II.5.10. In order to illustrate its use, assume that the design life of a dam is 50 years and that the designer wishes to take only a 10 per cent risk that the dam will be overtopped during its design life. Thus n equals 50, p_n equals 0.10, and the dam must be designed to withstand a flood that has a return period T_d of 475 years, which gives a probability of exceedance $p = 1/Td \approx 0.2$ per cent.

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